Bo Brampill

PRACTICAL INSTRUCTIONS

FOR CONDUCTING

TRIGONOMETRICAL SURVEY OPERATIONS.

BY

LIEUT.-COLONEL A. S. WAUGH, R.E.,

F. R. S., F. R. G. S., &c.,

SURVEYOR GENERAL OF INDIA AND SUPERINTENDENT

OF THE

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PRACTICAL INSTRUCTIONS

FOR

CONDUCTING TRIGONOMETRICAL SURVEY OPERATIONS.

GENERAL OBSERVATIONS.

1. The operations of the G. T. Survey of India are carried on in the form of a series of Principal Triangles, originating from, and terminated by, a well determined line,* and upon the sides of these Principal Triangles depend other minor ones, which furnish points at two or three miles asunder, thereby fixing limits to inaccuracy in topographical details.

2. The direction in which a series of triangles should proceed depends on the object for which it is undertaken, but, as a general rule, it is desirable to avoid an oblique direction, and to conform strictly to a meridional or longitudinal line; of these the meridional direction is the most favorable condition for geodetical surveys, because any error in the assumed figure and dimensions of the Earth has then least influence on the computed results.

3. Geodetical operations may be classed under two heads, viz.:—1st, Duties in the Field; 2nd, Duties in the Office. Under the first head are comprised the selection of the triangulation in a skilful manner, and the method of taking the final observations, with due regard to accuracy and precision. Under the second head are included the reduction of the observations and computations of the final results.

1st—*Duties in the Field.*

4. Although the measurement of a base line is an indispensable preliminary operation in a geodetical survey, and ought, therefore, as respects order of time, to be described first, still, as the Indian bases have already been measured, and the trigonometrical operations now carrying on in the British dominions in India, are all based upon the sides of other Series, it would be needless to lay down any rules for base line measurements. Should any necessity arise for undertaking the measurement of another line, the duty will be executed under the personal superintendence of the Surveyor-General of India.

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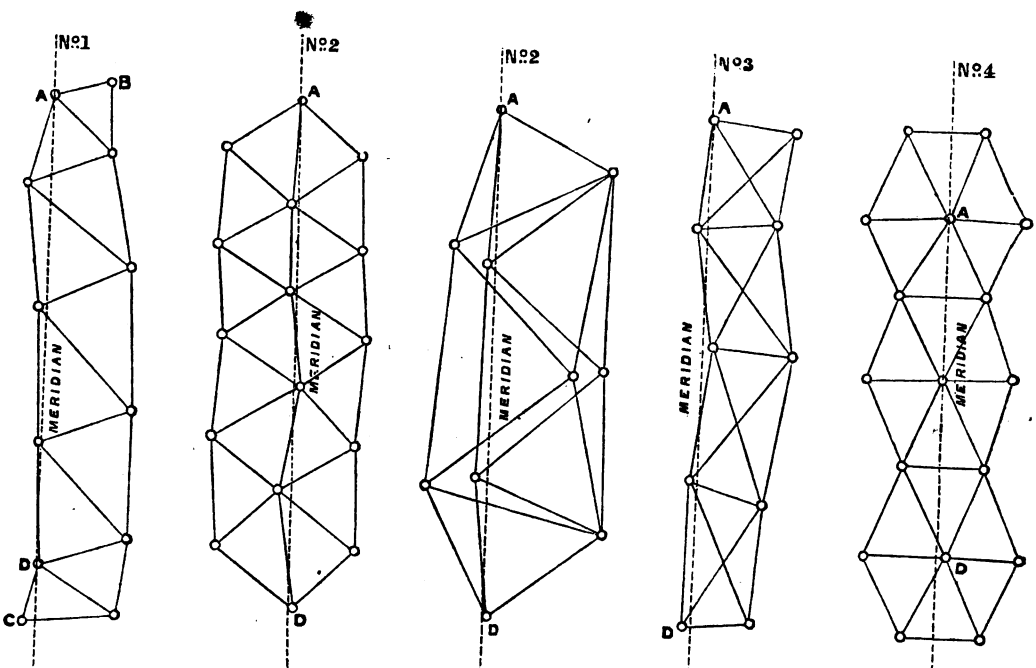
* The Great Arc and the several Longitudinal Series, which are considered the main operations of the Survey, depend on measured bases. All the other Series depend on sides of the Great Arc or Longitudinal Series, (excepting the Calcutta Meridional and the Great Indus Series, which, likewise, connect measured bases. The two last mentioned Series have been executed subsequently to the writing of this paper.)

5. The next important operation in conducting a series is to determine the direction of the meridian. There are several ways in which this may be accomplished, but the method most appropriate for geodetical purposes is to measure the azimuthal angle between a fixed terrestrial mark and a circumpolar star at the time of its greatest elongation. It will be most convenient to describe this operation under the head of Final Observations. A knowledge of the approximate position of the meridian is, however, indispensable in selecting the triangulation, but the data for this purpose are generally furnished by the prior Trigonometrical Series, upon which those now in progress are based, or, at any rate, the azimuth can always be determined with sufficient accuracy for the proper selection of stations, by means of a few observations on the Sun a little after sunrise, or before sunset. This approximate method is described in all elementary books, and an example is given in the Appendix, marked

Selection of Stations.

6. The direction in which the triangulation is to proceed being known, the next step is to select the stations in a judicious manner, so as to produce the greatest accuracy in the results.

7. A series of Principal Triangles may be either single or double, as represented below, wherein No. 1 is a single series. A double triangulation may



A AZIMUTH STATION. D AZIMUTH OF VERIFICATION. AB. BASE. D.C. BASE OF VERIFICATION.

be arranged in several ways; for example, No. 2 is a continued double series, No. 3 is a series of successive quadrilaterals, No. 4 is a series of successive polygons. In practise it frequently happens that all these varieties of figure are combined together in various ways, according to the nature of the country, and the local facilities it affords, but as each method has its peculiar advantages and disadvantages, it is proper that their relative merits should be duly considered, in order that the most judicious selection may be made which circumstances admit of.

8. The single series possesses advantages of economy of time and labor and money as well in the field operations as in the office computations; and it is, consequently, the most proper kind of series to be adopted when circumstances impose a strict attention to these restrictions. For instance, in a level country, like the plains of Hindustan, in which, from the absence of natural elevations, trigonometrical operations are both costly and tedious, every consideration of economy combines to recommend a single series as the most eligible arrangement. On the other hand, the advantage of accuracy will always be found greatly in favor of a double series, which, moreover, supplies a check of the most efficient kind at every stage of the work. The continued double series (like No. 2) is chiefly objectionable, because no part of the computations can be finally completed until the field operations are brought to a conclusion, under which state of circumstances the accumulation of arrears of office duty becomes a source of the greatest embarrassment.

9. This evil may be avoided by that arrangement of a double series, in which it is composed of a succession of quadrilaterals or polygons, because the computation of each figure can then be brought up independently from time to time, as the observations in the field are successively completed.

10. Of all geodetical figures the quadrilateral is theoretically the most conducive to accuracy, as well as most economical, because it requires no additional stations, and the equations from which the errors are deduced are proportionably the most numerous. For example, the relative conditions of the quadrilateral and of the usual polygons are as follows:—

	No. of Stations.	No. of Angles.	No. of Eqtns. formed.
Quadrilateral,	4	12	9(1)
Hexagon,	6	18	8
Pentagon,	5	15	7

11. These circumstances are greatly in favor of a quadrilateral, both as regards economy and precision; but, on the other hand, there are practical considerations of importance, which may frequently render the polygon arrangement preferable.

12. First, it may be remarked that it is extremely difficult to select four

stations forming symmetrical triangles, and at the same time so situated, with respect to each other, that from each station the other three shall be distinctly visible. Secondly, the long diagonal rays are unfavorable equally to good observation and to rapidity of progress. The diagonal of a square bears a proportion to the side of the same figure, in the ratio of $\sqrt{8}$ to 2, and it is obvious that in practice one diagonal of a quadrilateral will generally exceed that proportion. This combination of long and short sides is therefore liable to cause detention, on account of the difficulty of observing a distant station under circumstances which form no impediment to the observation of near ones. This embarrassment, however, is not of so serious a nature in a hilly tract as in a level country, where the difficulty of observing long rays is very greatly increased by the density of the atmosphere near the ground, and as, in such circumstances, towers are required to render these stations mutually visible, these structures will become a source of greatly enhanced expense. The curvature of a globular body increases in the ratio of the square of the distance, and it is therefore obvious that the height of a tower adapted to a quadrilateral is to the height adapted to a single triangle, in the ratio of $(\sqrt{8})^2$ to 2^2, or 8 to 4. Hence, a quadrilateral arrangement is totally inapplicable in a flat country, but in a mountainous or hilly region, quadrilaterals should always be chosen when the configuration of the country and other circumstances offer no practical objection of an insuperable nature.

13. A Meridional Series, as its name implies, follows the direction of the meridian of some fundamental station (*A*, for example, *vide* fig. 1), at which station the direction of the meridian must be duly determined by appropriate observations* on circumpolar stars. When the Series is composed of single triangles, or of quadrilaterals, it is advantageous to select all the stations on one flank, in such a manner as to coincide as nearly as practicable with the meridian. Those stations which are situated nearest to that line are the most favorable for observing azimuths of verification, which should be taken near the termination of the series, and also at intermediate points, fifty to eighty miles apart, according to the magnitudes of the triangles, because, the larger the triangles are, the less chance there is of accumulation of error, and as the azimuths of verification are intended to detect accumulated angular errors,[(2)] it is clear that the distance between the azimuth stations ought to depend on the number of intermediate triangles. When the Series is composed of polygons, the central stations of those figures should be selected on the meridian, or as near to it as possible.—*Vide* (fig. No. 4.)

14. In a Longitudinal Series, on the other hand, it is desirable that one flank of the Series (or the centres of the polygons) should coincide, as nearly as circumstances will permit, with the parallel to which the Series is intended to conform, and azimuths should be taken at every alternate station, in order that the latitudes and longitudes may depend on observed azimuths alone, and not

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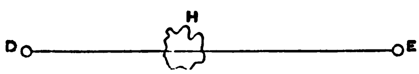
* The method of taking these observations and computing them will be explained hereafter.—*Vide* Para.

upon deduced ones, whereby the formulæ for computation will be confined within the proper limits.[3]

15. In order that the observed azimuths may be as free as possible from those errors which are caused by local attractions, it is desirable that no great mass of high mountains should stand either east or west of the station of observation, nor be so situated, with respect to it, that the portion of resolved attraction in the direction of the parallel should amount to a sensible quantity.

16. In choosing stations it is proper to avoid intermediate obstacles situated on, or close to, the ray, because if the ray between A and B grazes the side of the mountain C, the ray will certainly be deflected by the vapours which arise from the sloping ground, whereby the observations at A and B will be more or less injuriously affected by lateral refraction; and if the ray grazes close over the top of the mountain H, then the angles horizontal and vertical will be greatly disturbed. If circumstances, however, concur in admitting of no other choice, it will be desirable always to provide a check by introducing a polygon or quadrilateral at that part of the work, and a secondary station should also be established on C or H, which may be treated as an auxiliary point, for taking vertical angles, whereby the comparative height of A and B may be determined independently of direct observation.



17. A Principal Series should consist of triangles as large as the features of the country admit of, by which arrangement the number of stations will be reduced to a minimum; moreover, as the errors which are unavoidably committed at each station have a tendency to accumulate in proportion as those stations are multiplied, it follows that the probability of accumulation will be least when triangles are large.

18. Considerations of economy likewise impose the same restriction, inasmuch as the additional points required for topographical purposes can be obtained at less cost by means of secondary triangles, in which accumulation of error is an evil that need not be dreaded, the limits of inaccuracy being sufficiently established by the principal triangulation. No general rule can be applied with respect to the magnitude of triangles, which must necessarily vary with the configuration of the surface of the country. In a mountainous region with gigantic features (composed of lofty peaks and deep valleys) it would be as difficult as it would be injudicious to select small triangles; and in a flat country it would be absurd to struggle against natural difficulties in the vain endeavour to establish a series of great magnitude. In hilly countries, twenty to thirty miles is a convenient distance for principal stations, and such distances can usually be observed with facility, in all ordinary conditions of atmosphere, by means of heliotropes and

Argand lamps. When hills are table-topped, as is generally the case where the stratification of the rocks is horizontal, it may be necessary to shorten the sides to less than twenty miles. While, on other occasions, an open country with detached peaks may frequently afford distances of thirty to forty miles with the fairest prospect of good observations. In a level country, such as Bengal, ten miles is the most favorable length of side, and the limits should there be considered eight to fifteen miles.

19. The triangles selected should be symmetrical, that is, as nearly equilateral as possible, such being the best condition for ensuring accuracy when all the angles are measured, but as it is impracticable to obtain in the field triangles exactly equilateral, the rule in practice is, to admit of no angle under 30°, or above 90°. This rule applies to all the triangles which compose a series, whether principal or secondary; but, in the case of secondary triangles which do not form part of a long series, wider limits are admissible. These limits must also of necessity be extended in the case of a quadrilateral, because the four triangles of which it is composed obviously cannot be all equilateral. The best condition for that figure is when it approximates to the square, in which form no angle will be much above 90°, and none much less than 45°.

20. In a hilly region it is a convenient arrangement to detach a well qualified sub-assistant in advance, to choose the stations, and construct platforms, but in a level country it is advisable that the officer in charge of the work should select the stations in the early part of the season, leaving assistants to erect towers, and clear the rays in rear. Towards the end of the season he should return to the commencement, and begin final observations, while the assistants continue the clearing of rays, and erection of towers previously selected in advance.

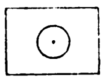
21. These arrangements however depend entirely on the organization of the party and the peculiar capabilities of the members who compose it; but, whatever plan is adopted, with regard to the Approximate Series, the following rules should always be adhered to :—

22. The stations selected should be on the highest peaks, but as these are sometimes inaccessible, it may be necessary to adopt a lower point. Every effort should, of course, be made to reach the summit when practicable. In the case of a lower point being used, care must be taken that the view is clear in the direction of the stations in advance.

23. The first business on arriving at a principal station, where points in advance are to be chosen, is to put up the instrument as soon as the atmosphere is clear enough to allow a distinct view. Having adjusted it, you will find out the direction in which you would wish the stations in advance to be, and by means of the level you will judge what points near these directions are likely to be visible from each other, and from the stations with which they are to be connected.

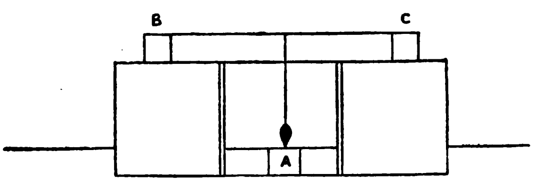
24. To avoid the necessity of revisiting the station from which the selection is made, it will in dubious cases be found advantageous to select more than one point from each station in advance, and when you are quite decided as to your choice, the platforms may be constructed.

25. It is the practice to mark all spots where angles are taken, whether they be principal or secondary stations. The mark is a dot with a concentric circle, cut on stone by means of a pointed chisel. If the mark can be engraved on the rock *in situ* so much the better; otherwise, a large stone, properly marked, ought to be buried in the ground. Over this a small platform is raised, on the summit of which another mark-stone is inserted, and fixed truly vertical over the lower one. The distance between the two marks should be recorded, but all measurements and observations are usually referred to the upper mark, and are so recorded in the angle book records.



26. Sometimes, on account of intervening obstacles, it is necessary to raise the platform to a considerable height, in which case several mark-stones are always inserted, and their relative heights recorded.

27. The method of adjusting mark-stones is as follows:—Let *A* be the mark at the foundation, either engraved on the rock, or on a heavy embedded stone. Let the external part of the platform be built up to the intended height of the next mark, and place upon it four heavy stones, arranged in a quadrilateral



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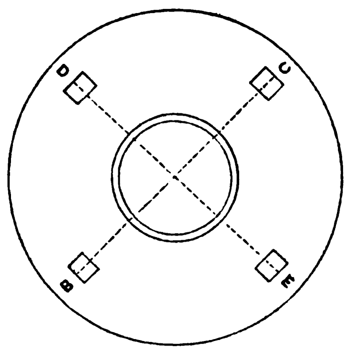


PLAN

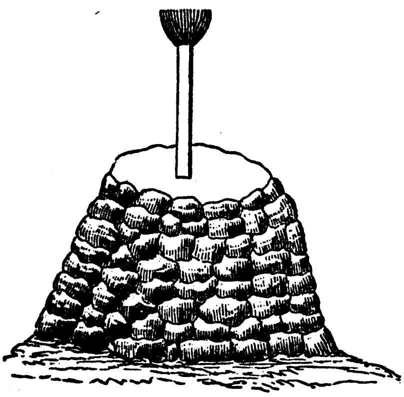
figure, in such wise that threads *B C* and *D E* stretched diagonally across may intersect near the centre. Adjust these threads to correspond with a plumb line suspended over the mark *A*, and when the coincidence is complete, mark the four exterior stones, by pencil lines, or lines scratched with a knife. Arrangements must now be made for protecting these stones, either by covering them over, or appointing a man to watch each, while the pillar in the centre is being built up nearly to the level of *B C*, at which height the next mark-stone should be fixed and adjusted, to correspond with the cross threads before adverted to.

28. For Principal Stations it is necessary to make that part of the platform on which the instrument rests, separate and distinct from that on which the observer and his assistants walk. The instrument is then said to be duly isolated. Unless this precaution is taken, good angles cannot be expected, as the instrument will be liable to irregular disturbance, according to the position of the observer. The annular space between the observer's stage and the central pier should be filled up with gravel or sand, otherwise screws and other parts of the apparatus may be lost, by falling into it. Besides the upper mark-stone, it is usual to imbed in the pier three picked, flat, heavy stones, for the tripod of the instrument to stand upon. These are called "feet stones," and they should be duly levelled, which will save trouble afterwards in adjusting the instrument.

29. It is not usual, if it can be avoided, to make isolated platforms at secondary stations. In localities where the ground is very unsteady, such as deep, black, cotton soil, I have found it practicable to steady a theodolite by using pickets four or five feet long, and driving them into the ground, for the stand of the theodolite to rest upon. The pickets isolate themselves for at least one foot in driving. This precaution can only be taken at a new station, otherwise the mark would be disturbed by the process of driving the pickets.

30. As soon as all the observations have been taken at any station, and it has been observed from all the corresponding stations, it is no longer required for the purposes of the Trigonometrical Survey, and it should have a pile of stones, with a pole and brush, erected over it, in order that it may be visible, and useful, to the detail surveyors. This precaution has the further advantage of protecting the mark-stones from violent disarrangement.

31. The pole and brush is erected thus: a long straight pole is selected, upon the top of which a brush of twigs is fastened. The pole is placed truly perpendicular over the Station mark, and a pile of stones raised round it, by means of which it is securely fixed. The diagram in the margin will give a clear idea of the signal, which is a very economical and useful opaque object for day observations. The pile of stones may be five or six feet high, and the pole about as much longer.

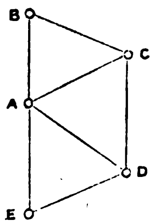


Final Observations.

32. The selection of stations is merely preparatory to the final observations, and is called an Approximate Series. It is usual to choose a few triangles in advance, and then detach a party to continue the Approximate Series, and pre-

pare the stations ahead, while the final observations are progressing in rear. This division of labor is of great advantage in accelerating progress, and, if judiciously arranged, with regard to local circumstances, and the organization of the party, is productive of the best results.

33. The angles at a station are taken thus: Supposing the Observer at *A*, and the signals at *B C D E* are all visible, the instrument is carefully levelled and adjusted, and so fixed that some station, *B* for instance, reads 0°, or zero; *B* is then called the zero station. Suppose the telescope to be brought up from the left hand of *B*, and turned gently, so that *B* may enter the field of view, and come near the centre wire, but not pass over it. Then clamp the instrument, and complete the bisection of *B*, by using the tangent screw of slow motion. Read off all the micrometers, or verniers, and let your assistant record the readings, in a fair legible hand, in the Angle Book. Look again into the telescope, and see that *B* remains bisected. If found correct, then carefully unclamp, and move the telescope gently towards *C*, taking care not to overshoot it. Clamp, bisect, and read off as before, and so on for *D* and *E*. You will thus have a complete set of observations at zero 0° observed by a continuous motion of the instrument from left to right. Now, overshoot the station *E*, and bring the telescope back by a continuous motion from right to left, observing each station in succession, and recording the readings. This will give a second set at zero 0°, which, if accordant, will suffice for that zero. It is the practice of the Trigonometrical Survey to make at least one repetition, in order that if mistakes creep in they may not pass undiscovered.



34. Now turn the telescope over 180° in altitude, and round 180° in azimuth, so that if the face of the vertical circle were previously to the left hand, it will now be to the right hand; *B* will then read 180°, and this is called zero 180° *F R*, the former position being zero 0° *F L*, *i.e.*, face left. Proceed as before, and take ~~two~~ 3 sets of observations, the motion of the instrument being in one set continuous from left to right, and in the other from right to left, as before.

35. Having thus obtained ~~four~~ 6 sets of observations for one position of the instrument, you will now, if the number of microscopes be three,* bring 10° under micrometer *A*, at which position you will get four other sets, and so on for 20°, 30°, &c., up to 50°, whereby you will obviously bring every 10° of the circle under one or other of the microscopes.†

* The Great Theodolites have five microscopes, and observations are taken at every nine degrees of the limb, by means of four change of zeros; the 18-inch and 24-inch have three microscopes.

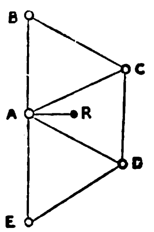
† Since this was written Major Walker has suggested that although this method of changing zero gives readings at equal parts of the limb round the circle, whereby the errors of graduation may be eliminated, there is no great change in the position of the axis, the

36. By this process you will have six changes of zeros and twenty-four readings, which will suffice for a complete set of angles, and you ought not to be satisfied with less, when carrying on work of primary importance. All operations in which these precautions are not duly taken are considered to be of inferior order. This rule applies also to observations for azimuth taken upon circumpolar stars, which must always be taken at six zeros, or at every 10° of the limb at each elongation, as will hereafter be explained.

37. I have supposed, in the foregoing instructions, that the signals at all the stations are simultaneously visible, which is not always the case. It is generally so when lamps are used at night, but when heliotropes are employed it is evident that an eastern station will be seen with difficulty in the morning, whereas, in the evening, a heliotrope will shine vividly, and *vice versâ* in the case of a western station. Under these circumstances, the observations cannot be taken in regular rounds, in the simple manner before described. The best plan in this case is to use a point of reference, and connect each station therewith at such times as may be most convenient for observation.

38. The site selected for a referring mark should be such, that it may be visible at all times, its distance from the Principal Station should be from one and a-half to three miles, and it should be nearly on the same level, in order that the elevation of the Telescope may be as little disturbed as practicable during the progress of the observations. It is also convenient that the situation of the referring mark should be central with respect to the circumjacent stations which have to be connected therewith.—*Vide* figure in the margin.

39. In taking angles with the referring mark, it is often the case that the sets are observed by parts, but as it will not do to compose an angle of two parts taken at different divisions of the limb; therefore, you must be careful in taking each supplemental part, to bring successively under microscope A the divisions which stood there when you observed the first part.[4]



These two distinct parts, when assisted by addition or subtraction, may then be treated as a whole angle; for example, the angle $BAC = \text{the} \angle BAR - \text{the} \angle CAR$, and if the former angle be observed at a different time from the latter, then the reading of the point R should be the same, within a few seconds, at

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defects of which may still affect the results. He proposes to obviate this by adding the arc between the microscopes to the arc by which the zero is changed. Thus, in an instrument with five microscopes for five changes of zero the instrument will be set successively to

(0°–1′) (79°–12′) (158°–24′) (237°–36′) (316°–48′).

and for a three microscope instrument with six changes the zero will be set successively to

(0°–1′) (70°–12′) (140°–23′) (210°–34′) (280°–45′) (350°–56′).

By this method the axis will make nearly a complete revolution during the set of observations, and each angle will be observed at equidistant parts of the axis, as well as of the graduation. This alteration is therefore judicious, and should be carried out in future practice.

both times, so that the deduced angles may be composed of entire arcs of the limb of the theodolite, in the same manner as if they had been directly observed without the intervention of a referring mark. If there be among the principal stations any one so favorably situated as to hold out every prospect of being at all times visible, it may be treated as a point of reference, and the other stations may, accordingly, be connected therewith, which will save the trouble of observing a supplemental point.

40. It may here be remarked, that though a referring lamp is sometimes used for the deduction of angles, still it should be resorted to only on occasions of difficulty, because the angles taken in parts are not likely to be so accurate as those taken directly. If e be the probable error of any single direct angle, then a deduced angle, composed from the sum or difference of the two observed angles, will be liable to an extreme error of $2e$; therefore, when all the points are visible, it is best to observe in whole rounds, because, independent of the saving of labor, the errors of contiguous angles do not then enter into combination with one another. Thus, on reference to the fig., it will be seen that $\angle\ B\ A\ E$, which is composed of three angles, $B\ A\ C$, $C\ A\ D$, $D\ A\ E$, is not liable to be burdened with a greater error than any one of the component angles, provided the observations are taken in rounds.

41. Having described in a general manner the method of observing pursued in the Great Trigonometrical Survey of India, it is necessary now to remark upon the chief precautions necessary to be taken, in order to ensure the greatest degree of accuracy, which the means at our disposal admit of.

42. The observer ought to be skilful, scrupulous, and patient, and should cultivate, by incessant practice, the powers of vision, and delicacy of handling instruments.

43. The instrument should be kept in good order, and handled with the greatest care. Separate instructions on this head will be given hereafter.

44. The stability of the foundation and due isolation of the instrument are essential elements of accuracy, as already remarked in para. 28. The instrument should also be carefully centred over the station mark, and made truly level, for effecting both of which purposes appropriate apparatus is attached to every instrument. Levelling with three feet-screws is very easily and efficiently performed, because three points define a plane, and it is clear that if the instrument be levelled in one direction by two feet-screws, the remaining foot-screw being at right angles to the line joining the former, completes the adjustment of the instrument in both planes.

45. In pursuance of this principle, let the level of the body of the instrument be placed parallel to two feet-screws, and, by means of those, bring the bubble to float in the centre of its tube. Then turn the instrument round 180° in azimuth, and if the bubble continues to float in the centre, the instrument must

be truly level in that direction, otherwise half the difference of the reading at each end of the bubble is the error, which ought to be corrected by the feet-screws, and the other half difference of the reading is the error of the level itself. The latter error need not be corrected at all, if it amounts only to a few divisions, because it can always be allowed for in performing the adjustment. For instance, if the right hand end of the bubble reads a divisions, and the left hand end reads b divisions, in one position of the instrument, and after a semi-revolution in azimuth the ends of the bubble read a' & b', then $\frac{a+a'}{2}$ and $\frac{b+b'}{2}$ are the readings which the bubble ought to have when the instrument is truly level. Now turn the instrument round 90° in azimuth, and, by means of the third foot-screw, bring the bubble to read $\frac{a+a'}{2}$, $\frac{b+b'}{2}$. The instrument will now be approximately levelled, and if this operation be repeated two or three times, the adjustment will be perfected. If the error of the level be very large, that is to say, if the reading of one end of the bubble differs considerably from the reading of the other, it can be rectified by the small capstan-headed screws attached to the level for that purpose. The proper time for effecting the practical correction is when the instrument is nearly level; but if the error be small, it is better, as before stated, to leave it alone, and allow for it in levelling, because the materials of the instrument have a tendency to settle into a position of equilibrium, in which they will remain steady, but, if frequently disturbed, the screws will in time work loose, and no confidence can be placed in the permanence of the adjustments.

46. The apparatus attached to the level for the purpose of adjusting it in due relation to the axis of the instrument, is very different in various instruments; for, in some, there may be a single screw at one end for raising or depressing that extremity of the level; in other instruments there are two antagonizing screws at one end of the level, and one must be released before the other is tightened.

47. In most modern instruments, however, there are three screws at each end, of which the central of each set it a drawing screw, and the two external ones are pushing screws. In this case, if one end of the level requires lowering, release the central screw, and tighten the external ones. On the contrary, for raising that end of the level, release the external, and tighten the drawing screws. It is a general rule, in working antagonizing screws, that is to say, screws that produce opposite results, that one should be released before the other is tightened, otherwise the screws are liable to be bent, or the threads broken, and the efficiency of these delicate instruments to be completely destroyed.

48. The criterion of the instrument being level, is, that each end of the level reads the same during a complete rotation in azimuth. The accuracy of the adjustment should therefore always be tested before final angles are taken, by reading the level in a position parallel to two feet-screws, and then successively at

90°, 180° and 270° in azimuth. A difference of about 1 division may be considered of no account in taking terrestrial angles, but one cannot be too scrupulous in levelling the body of the instrument when stars, or other elevated objects, are observed.

49. The process of levelling is liable to disturb the centering of the instrument over the station mark, which should therefore be duly looked to. Instruments are centered either by means of a plummet, or by means of a look-down telescope.

50. The point of suspension of the plummet may, perhaps, not coincide precisely with the centre of the axis; therefore, if the plummet revolves with the instrument when the latter is turned round in azimuth, the centre will not agree after a semi-revolution, and must be corrected for half the difference. If the point of suspension of the plummet does not revolve with the axis of the instrument there is no method of correcting the eccentricity, which is therefore irrememediable.

51. When a look-down telescope is employed for centering, the instrument must first be duly levelled, the eye piece of the look-down must be adjusted to distinct vision of the wires, and the object glass adjusted to distinct vision of the station mark, so as to be free of parallax. Now, by means of the centering screws, move the instrument till the station mark is duly bisected by the cross wires. Then turn the instrument round 180° in azimuth, and if the station mark is seen on one side of the wires, half the difference is the error of centering, the remainder being the error of collimation of the wires. Adjust the wires accordingly, and also the centering, after which relevel, and repeat the process, which will generally suffice to perfect the adjustment. The advantage of a look-down telescope over the plummet is two-fold;—first, it is not disturbed by currents of wind; secondly, the adjustment is more minutely performed, by reason of the magnifying powers of the telescope.

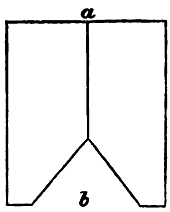
52. The body of the instrument being duly centred and levelled, the next care of an accurate observer is to render the transit axis truly horizontal. The telescope of every theodolite, with its attached vertical circle, is supported on a horizontal axis, the ends of which, called "pivots," rest in angular supports, termed Y's.

The imaginary line, joining the centre of the pivots, is the axis on which the telescope rotates, and, in order that the latter may describe true verticals, the axis must be truly horizontal. This adjustment is effected by raising or lowering one of the Y's by the appropriate screw, placed beneath the Y for that purpose,—the amount of adjustment being regulated by the indications of a riding or striding level, which must be placed upon the pivots during the process of levelling them.

53. The riding level is furnished with two feet, which are cut into the shape of notches, or inverted Y's, and there is either a single screw or two antagonizing

screws at one end of the tube, whereby to adjust the bubble to parallelism with the line of bearing of the feet. There is also generally attached to one end of the riding level a little cross level, by means of which the inverted feet **Y**'s are made to rest always with the same points on the pivots, and any slight discrepancy in the verticality of the feet **Y**'s is thereby prevented from having any effect.

If the figure in the margin represent an inverted foot **Y**, and *a b* be the line bisecting the angle of the **Y**, then it is clear that if *a b* be truly vertical, each side of the **Y** will rest equally on the pivot; but if the similar line *a′ b′* of the other foot be not truly parallel to *a b*, then, on a slight declination of the line *a b*, the level will rest on the pivots by three points only, instead of four, and the bubble will, by reason of the obliquity of the **Y**, run up towards one end. The remedy for this evil is to render the two feet of the level truly parallel with respect to the imaginary lines bisecting the inverted **Y**'s. For this purpose release the screws which attach the feet to the tube of the level, and gently turn the feet until, by successive trials, it is found that the bubbles remain stationery, even when the level is inclined a little on one side or the other of true verticality, as respects the bisecting line of the **Y**'s. The proper bearing on the pivots will thus be maintained, and the riding level will give true indications, even without the aid of a cross level.



54. To level the transit axis—Uncover the pivots by removing the clips which retain the pivots in their **Y**'s. Introduce the riding level gently between the radii of the vertical circle, and place it upon the pivots. Adjust it to the cross level, if it have one, and read off the ends of the bubble. In order to distinguish one end of the bubble from the other, it is usual to call the end next the little cross level *L*, and the opposite end *O*; but if there be no cross level, it is as well to mark the letters *L* and *O* on the tube, as characteristic marks. It is also usual to call the divided face of the vertical circle *F*, and the other face *P*. Now, suppose, in the first instance, that *L* end of the level is at *F*, and the readings of the bubble at that position are *l* and *o*, gently take off the riding level, and reversing it, end for end, replace it on the pivots, so that *L*, which before was at *F*, will now be at *P*; read off the bubble, and let the readings be *l′* and *o′*, then half the difference between the reading of the same ends in the two positions will be the inclination of the transit axis, to be adjusted by the proper screw under one of the **Y**'s, whereby that **Y** may be raised or lowered. If $\frac{l - l'}{2}$ be greater or less than $\frac{o - o'}{2}$, the difference is occasioned by unequal expansion of the level, and the mean must be adopted as the true error. The other half difference of the readings is the error in the level itself, which, if it amount to any considerable quantity, may be practically corrected by the appropriate screws attached to the level.[5]

55. It is clear that the foregoing process only levels the upper surface of the pivots; consequently, if the pivots are unequal in diameter, their centres, which form the true axis of rotation, will not be level. The inequality of the pivots is generally so minute as to be a rejectaneous consideration; but, if it happens to amount to an appreciable quantity in any case, there is no doubt that it could be measured and allowed for in levelling the axis. For this purpose all that is requisite is, to make the pivots change places in the Y's, without reversing the level, and one-fourth the difference of the readings of the bubble before and after changing pivots is the amount of error due to this inequality, which may be allowed for by making the larger pivot, or that to which the bubble ascends on reversal, always read higher by one-fourth the difference. Although this experiment is simple enough in principle, it is difficult of execution, requiring great delicacy of manipulation to reverse the pivots without disturbing the instrument; fortunately, the inequality is inappreciable, and may safely be neglected in practice.

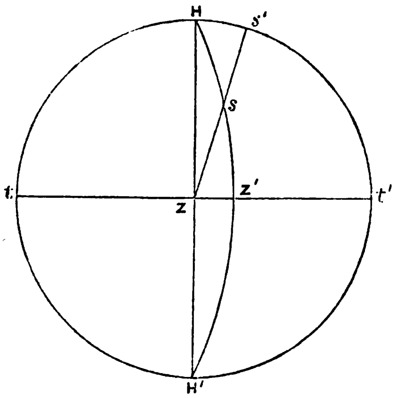
56. It is inadvisable to alter the inclination of the transit axis for an error amounting only to four or five seconds, because frequent tampering with the screws will in time loosen them, and destroy the permanency of the adjustments, the stability of which is a most important circumstance in making observations, because what will it avail to render an instrument free from error at the commencement, if the adjustments do not continue permanent throughout a set of observations.

57. The value of the divisions of the scale of a level may be ascertained by affixing the level to the frame of the vertical circle, or making it ride parallel to the telescope, and then taking the readings of the microscopes in two positions of the bubble, whence comparing the number of divisions of the level scale run over by the bubble with the corresponding angular motion of the vertical circle, as measured by the microscopes, the value of one division of the level scale will be obtained by simple proportion, as shown in the following example:—

Experiments made at Kalianpur, January, 1840, to determine the value of the divisions of the axis level appertaining to the circle Troughton.

Temperature.	Vertical Microscopes.			Angular Difference.	Level.					Computed value of one division of the Level Scale.
					Readings.		Differences.			
	D.	E.	Mean.		L.	O.	L.	O.	Mean.	
Farhnt. 70·7	7 25 47·0	41·0	7 25 44·00		85·6	48·4				
	7 26 20·5	14·1	7 26 17·30	0 0 33·30	50·9	83·0	34·7	34·6	34·65	0·9610
	7 26 12·1	6·0	7 26 9·05	0 0 8·25	59·9	73·9	9·0	9·1	9·05	0·9116
	7 25 62·1	56·2	7 25 59·15	0 0 9·90	70·1	63·8	10·2	10·1	10·15	0·9755
	7 25 52·2	45·0	7 25 48·60	0 0 10·55	81·0	53·1	10·9	10·7	10·80	0·9769
	7 25 44·2	37·3	7 25 40·75	0 0 7·85	89·9	44·3	8·9	8·8	8·85	0·8870
	7 26 11·1	4·9	7 26 8·00	0 0 27·25	61·2	73·0	28·7	28·7	28·70	0·9495
										5·6615
									Mean value of one division of the level scale, ...	**0·9436**

58. If in the figure in the margin, which represents an orthographical projection on the plane of the horizon, Z be the true zenith, and $t\,t'$ be the direction of the transit axis, then, when that axis is level, the telescope will rotate in the vertical plane $H\,Z\,H'$ at right angles to $t\,t'$, and when the telescope is elevated 90° it will point to Z, the true zenith of the place. But if the transit axis be inclined to the plane of the horizon, the telescope will decline from the point Z by an angle $Z\,Z'$ = to the amount of the dislevelment of the axis. The telescope will now rotate in an oblique plane $H\,Z'\,H'$, and if S be an elevated object, the telescope will refer its position to H on the horizon, instead of to S', which is the true horizontal point appertaining to S, as determined by the true vertical $Z\,S\,S'$ passing through Z and S. The angle $H\,Z\,S'$, or corresponding arc $H\,S'$ is the error in azimuth occasioned by the dislevelment of the transit axis, and in the spherical triangle $S\,Z\,Z'$ right angled at Z', we have cos. $S\,Z\,Z'$ = tan. $Z\,Z'$ cot. $Z\,S$, or as the arcs $Z\,Z'$ and $H\,S'$ are both small, amounting in fact to a few seconds, we have—



$$H\,S' = Z\,Z' \times \text{tan. altitude.}$$

59. This formula clearly shows that when the altitude is 0°, an inclination of the transit axis produces no error in the azimuth readings; and also the error in azimuth must be very small when an object is little elevated, as is the case with terrestrial objects. It is also evident that the error lies in the direction in which the pivot is higher, so that if the left hand pivot be higher, the error will be *minus*, and corresponding correction *plus;* but, on reversing the face of the instrument, the pivots change positions with respect to the observer, and the errors will have an opposite sign; consequently, if the mean be taken between an observation face left, and another face right, the influence of an inclination in the axis will be entirely exterminated, and this will also happen with regard to depressed objects, in which the signs of the errors will be opposite to those in the case of elevated objects. Hence, if angles be taken between an elevated and a depressed object, those on a single face will be burdened with the sum of the errors due to the depression and elevation, but the mean of the angles between face left and face right will be free from error.

60. The next adjustments to be undertaken are those of the telescope itself, which are four in number, and all have reference to the wires, or ocular lines, situated near the eye end. These adjustments are,—

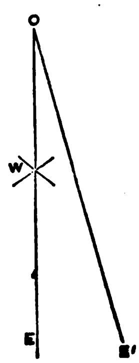
1st—Distinct Vision of the Wires,
2nd—Freedom from Parallax,

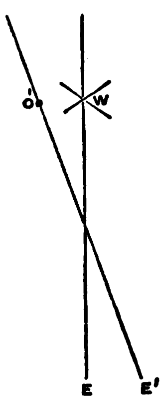
3rd—Collimation in Azimuth,

4th—Verticality of the Vertical Wires, and horizontality of the Horizontal Wire.

61. The only adjustment of a personal nature is that for distinct vision of the wires, which varies with the focus of the observer's eye, and the eye piece must therefore be pushed in or drawn out of its cell, until distinct vision is obtained. It is advisable to direct the telescope to the sky, or hold a piece of paper obliquely in front of, and a short distance from, the telescope, so that the wires being projected on a blank field, may be viewed by the eye, undisturbed by the contemplation of other objects. The wires will be truly in the focus of the eye piece when they appear sharply defined, and all the little specks of dust on them are seen clear and distinct.

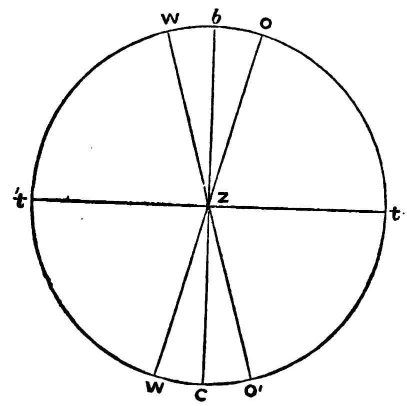
62. Now direct the telescope upon a distant object, a heliotrope, for instance, which bisect with the vertical wire. Move the eye gently to one side, and if the object still appears bisected there is no parallax, and the wires are truly in the focus of the object glass. If, however, on moving the eye to one side, the image of the object appears to move with the eye, then the focus of the telescope lies beyond the wires, and the object, glass and wires must be approximated to each other, because if *O* be the image in the telescope, and *W* be the wires, and *E* the position of the eye when *O* is bisected, then when the eye moves to *E′* the image will be seen to the right of the wires, and will appear to have moved with the eye, which is called far parallax. If, on the other hand, the image *O* is between the eye and the wires, then it will move contrary to the eye, because if *E* be the position of the eye when *O* is bisected at *W*, then when the eye moves to *E′*, *O* will be seen at *O′*. This is called near parallax, and may be rectified by augmenting the distance between the object glass and the wires, which adjustment is effected in some instruments by moving the object glass, in others by moving the tube which contains the wires. Small instruments usually have a pinion and rack to regulate the adjustment of the focus, but this is not the case with the larger class of instruments.

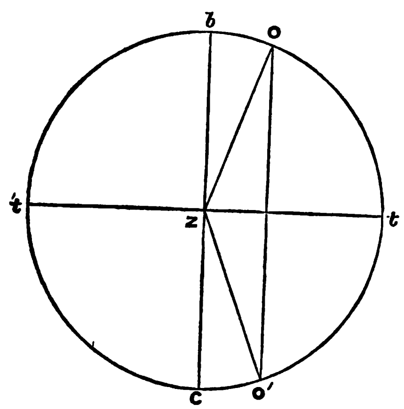




63. The line of collimation in a telescope may be defined to be that optical line which, passing through the centre of the wires and the centre of the object

glass, at the same time intersects the axis of motion at right angles. In a theodolite or alt.-azimuth instrument, the telescope rotates on the transit axis, and the line of collimation in a telescope thus mounted is the optical line passing through the centre of the object glass at right angles to the transit axis. If the wires do not coincide with the true line of collimation, the telescope will not describe true verticals. For, let *t t′* be the transit axis, *o w* the optical line, joining the centre of the wires and centre of the object glass, that is to say, the line of vision. Now, suppose this line to rotate round the axis *t t′*, and it is clear that it will not pass through the zenith at all, but after a semi-revolution the object glass *o* will arrive at *o′*, and the wires *w* will arrive *w′*. The false line of collimation will in fact describe a cone, the base of which will be a small circle parallel to the vertical, which intersects the axis at right angles. For, let *t t′* be the line passing through the axis, and *c b* the vertical at right angles thereto; let *z* be the projection of the zenith on the plane of the horizon, and *z o* be the direction of the line of vision; now, suppose the telescope to revolve round the axis *t t′*, the line *z o* will describe a cone of which the base *o o′* is a small circle parallel to *b c*, the great circle vertical to the horizon, and perpendicular to the axis *t t′*.

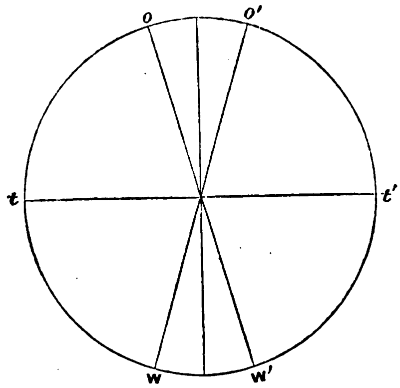




64. Having defined what the line of collimation in azimuth is, the next thing to be considered is, the practical method of adjusting it. There are several ways in which this adjustment may be effected, and of these the following is the one usually adapted in the field with moderate-sized instruments.

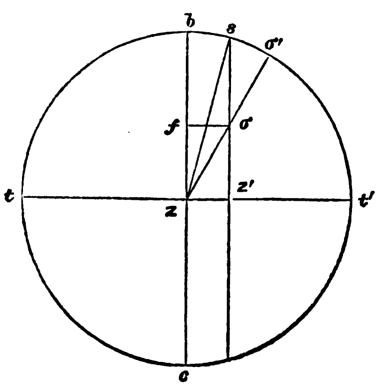
65. Direct the telescope to a distant fixed object, and bisect it with the wire. Then lift the telescope out of the Y's, and replace it with reversed pivots, so that the right hand pivot *t* may now be at the left hand *t′*. This must be

done with delicacy, taking care not to shake the instrument, as the success of the observation depend on the Y's remaining unmoved. If the line of collimation be true, the object will appear bisected in the second position of the telescope, otherwise it will be seen to one side of the wires, and half the apparent is the real error to be corrected by the appropriate screws at the eye end of the telescope. Two or three repetitions will suffice to perfect this adjustment, and when it has once been effected, it should not again be tampered with unless the error of collimation amounts to more than 3″ or 4″.



66. It is very clear that pursuing the process described in the foregoing paragraph the amount and direction of collimation can easily be measured on the limb of the theodolite, and may be used as an element of reduction, instead of practical adjustment. All that is necessary is, after intersecting the distant fixed object, to read off the microscopes, then lifting out the telescope, and replacing it with reversed pivots, again bisect the object, and read off the microscopes. One half the difference of the readings will be the error of collimation, and its sign may be known from the following consideration :—Suppose the instrument, in the first instance, to have stood face left, with the pivots in their usual Y's, and suppose the readings in that position to have been in excess of those obtained after reversal, then the error of collimation is +, as respects all observations on Face Left, and —, as respects those on Face Right. Consequently, in applying the reductions dependent on this error, the corrections will have opposite signs, viz., — for Face Left, and + for Face Right.

67. If the distant fixed object, by means of which the collimation has been determined is situated above or below the horizon, the collimation error thus obtained must be reduced by the cosine of the altitude or depression of the object observed. On the other hand, the amount of the collimation error at the horizon being known, the azimuthal error C' at any altitude, a for example, will be $C' = c \times \sec. a$ in which, $C =$ collimation error at the horizon; for, let σ be an elevated object (a star for example), the line of collimation $Z\sigma$ rotating on $t\,t'$ will refer σ to S instead of



to σ', which is the true place of the star as referred by a vertical circle to the horizon. Now $b\,s = f\,\sigma = z\,z' = c$ the error of collimation in azimuth at the horizon, $Z\,\sigma = a_{(1)}$ the zenith distance of σ, and $z\,f\,\sigma$ is a spherical triangle right angled at f, from which we obtain sin. $b\,Z\,\sigma =$ sin. $C' = \frac{\text{sin. } c}{\text{sin. } a_{(1)}}$ or as c and c' are very small,

$$C' = c \text{ sec. } a.$$

68. This formula clearly shews that an error of collimation produces its least effect on azimuth readings when the altitude of the observed object is nothing. It is moreover evident that the system of observing with the face of the instrument alternately left and right must necessarily eliminate the effect of the errors, because they lie in opposite directions on opposite faces.

69. The permanence of the direction of the line of collimation is a most important consideration, as any change during the course of the observations will prevent the errors cancelling each other. It is found in practice that the line of collimation is liable to a slight fluctuation of a few seconds, on account of the unequal expansion of the metal composing the telescope, but the errors incidental to this fluctuation have a tendency to compensate during a long series of observations.* In observing stars an erroneous collimation (like the dislevelment of the axis) produces an enhanced effect on the azimuth, according to the elevation of the object, and it is chiefly for that reason that the subject has been dwelt on at so great a length. To complete the subject it will now be necessary to describe other methods of adjusting the line of collimation, or determining its amount and direction.

70. The previously described method is that which is usually practised with moderate-sized instruments, but it was found inconvenient when applied to the large theodolites in use on the Great Arc. The extreme accuracy with which those instruments were graduated enabled an angle to be measured by a single determination to about 1″ from the truth. Hence the difference of the readings on opposite faces, as obtained by observing an object alternately face Left and face Right, gave double the error of collimation, the amount and direction of which could always be inferred with great confidence from the regular observations, and might easily be corrected, when required, by the appropriate screws at the eye end of the telescope. This method, however, does not appear to be applicable to engine-divided instruments, because the microscopes, on reversing the face, will stand 60° apart from their previous positions, thus embracing the greatest change in zero which the instrument is capable of, and introducing the full amount of uncertainty due to graduation.

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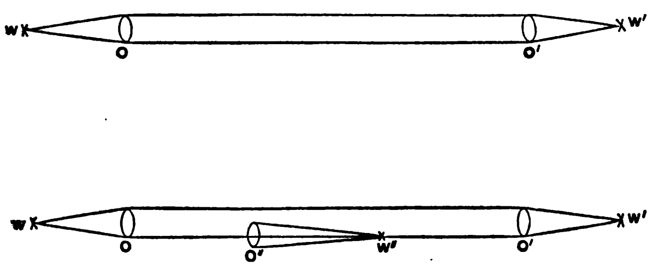
* As a precaution against unequal expansion, it is proper to clothe the telescope and axes with woollen cloth, and to protect the instrument from cold currents of air, &c., and from the prejudicial effects of solar rays.

71. Gaus's method of collimating is free from the foregoing objections, being equally independent of errors of graduation, and of the practical difficulty of reversing the pivots. Two auxiliary telescopes are requisite in this method, and the most convenient to use are those belonging to the small theodolites. It is an indispensable preliminary that all three telescopes, that is to say, the one to be collimated and the two auxiliary ones, should previously be adjusted to solar focus, which is easily effected by directing them upon some distant object, and eradicating parallax (*vide* para. 62). The definition of solar focus is, that parallel rays impinging on a lens will, after transmission, converge to that point, and conversily, if rays emanate from that focus, they will after penetrating the object glass proceed in a parallel direction; moreover, if the parallel rays be received on the object glass of another telescope they will, after passing through it, converge to its solar focus. Hence, if *o* and *o′* be the object glasses of two telescopes directed towards each other, and *w* and *w′* be their respective solar foci, in which let us suppose wires to be fixed, then it is clear that those wires will become mutually visible, because their images will be produced in the foci of the wires of each telescope, that is to say, the image of *w* will be apparent at *w′*, and the image of *w′* will be visible at *w*. Now, if a third telescope likewise adjusted to solar focus, be placed intermediately with respect to the other two, it is clear that the parallel rays emanating from the latter will render their wires visible in the intermediate telescope when it is pointed at either of them. The line *w w′* joining the wires of the two exterior telescopes is a right line to which all the other rays are parallel, consequently, if the intermediate telescope *o″* be made first to intersect *w*, its line of collimation will be parallel to *w w′*; and if the telescope to which *o″* belongs be made to rotate 180° in altitude, so as to point towards *w′*, the line of collimation will still remain parallel to *w w′*, provided it is truly adjusted at right angles to the axis of rotation; therefore, if on reversal, the point *w′* is not bisected, half the apparent deviation will be the error of collimation or deviation from perpendicularity to the axis of motion.



72. The practical execution of this method of collimating obviously consists in placing two small theodolites, one on each side of the instrument to be collimated, and in suchwise that all three instruments shall be nearly in the same straight line.* To effect this, set the telescope of the centre instrument level, and

* A few tenths of an inch are not important because it is the parallelism of the rays, and not the identical centre of the object glasses, upon which the method depends.

place a theodolite so that the wires may be mutually visible. Now turn the centre telescope over 180° in altitude, and place the other theodolite in suchwise that its wires may become mutually visible. Then remove the centre telescope, by lifting it out of the Y's, whereby you will be able to adjust the two external telescopes upon each other, so that their wires shall mutually intersect; after this they are not to be disturbed. Replace the centre telescope, and directing it upon one of the small theodolites, intersect the image of its wires, and then turn over 180° in altitude, so that the telescope may now point to the other theodolite; and, if the wires of the latter are found to be also intersected, the line of collimation is truly at right angles to the axis of rotation, otherwise one half the apparent is the true deviation to be adjusted, or measured by the microscopes, as the case requires. It is obvious that this method, with a little modification, may be made applicable to collimation in altitude.*

73. The distance between the telescopes is necessarily limited by the fineness of the wires and the magnifying powers in the auxiliary theodolites, the wires of which are not likely to be visible at a greater distance than ten or twelve feet; consequently, they must be placed about five feet from the telescope to be collimated, or at any distance greater or smaller, which on trial may be found suitable. To render the wires distinct, an assistant should reflect light into the eyepiece of the telescope whose wires you wish to see, and for this purpose a slip of paper held obliquely will answer very well.

74. The fourth adjustment in the telescope referred to in para. 60 consists in rendering the wires in the telescope perfectly vertical and horizontal. The system of wires or occular lines used in the focus of telescopes are very various,— No. 1, consisting of a single vertical crossed by a horizontal wire, is the best adapted for the principal triangulation, as the objects observed are generally round luminous points. No. 2 is better adapted for observing flag staves, spires, and other perpendicular objects, which are very clearly seen in the fork of the wires.

As the intersection of three lines, however, occupies a large undefined space, it is preferable that the horizontal wire should not pass through the intersection of the oblique lines, but form a small triangle therewith. In this case the part *a* is the best place for observing elevations and depressions, and the part *b* is best for azimuthal observations. No. 3 is a net as usually applied to astronomical circles. These wires are generally fixed at a proper angle on a perforated plate, which admits of a small play for the purpose of adjustment. To verify the horizontal

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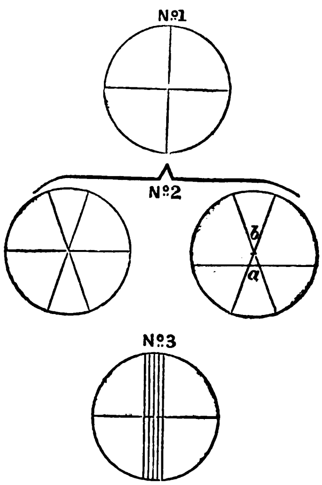
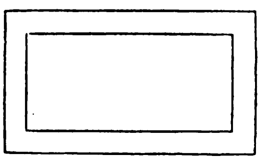
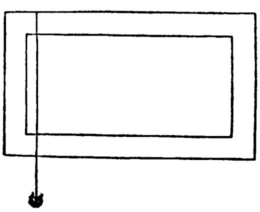
* For example: By means of the vertical tangent screw intersect the wires of one of the auxiliary theodolites, turn round 180° in azimuth, and the telescope will point to the other auxiliary theodolite. One half the apparent deviation in altitude will be the error of collimation to be adjusted by the screws acting on the wires. After which the micrometers should be adjusted to zero. This method is seldom employed, as with an altitude and azimuth circle the index error can easily be corrected by means of observations taken on both faces as will be explained in para.

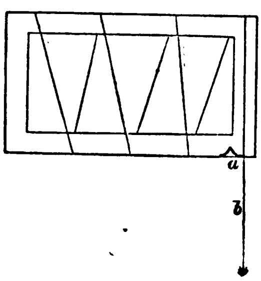
wire all that is necessary is to set the telescope to a distant steady defined object near the horizon, and move the instrument in azimuth, so that the object may appear to move along the wire from one extremity of the field to the other. The instrument should, of course, be previously well levelled. To adjust the vertical wire all that is necessary is to move the telescope in altitude, so that a distant object may appear to traverse or run along the vertical wire. If the object do not remain intersected at all parts of the field, the error ought to be rectified by moving the wire plate in the appropriate direction. The relative position of the wires, with respect to each other, having been fixed by the maker, it is only possible to adjust one wire by means of the wire plate, but if there be a micrometer at the eye end, it can generally be adjusted separately.



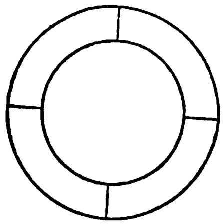
75. The wires should be stretched tightly, for if they be festooned their intersection will not remain permanent during the revolution of the telescope on its axis. As wires are liable to be broken, to become flaccid by damp, or uneven by accumulated dust, it is necessary that every Surveyor should be able to replace them when required. The best substances to apply to large instruments are spider's lines, and to small ones, the fibres of raw silk. To procure spider's lines prepare some card frames, by removing the internal parts, as shewn in the margin. Next look out in the garden, or among trees and bushes, for some healthy spiders, and take one up on the edge of the card frame. Then gently shake the frame, to detach the spider, which will hang therefrom, as shewn in fig. 2. Wind up the fibre, so that it may be rather widely apart, *vide* fig. 3. When you have come to the end of the card make a notch (*a*), in which insert the end of the thread, and then cut off the rest with a pair of scissors. If you wish to cover more cards, do not let the spider fall to the ground, where it would be covered with dust, but lay hold of the line at *b*, and place it in the notch of another card, which wind up as before. The

N°1


N°2


N°3


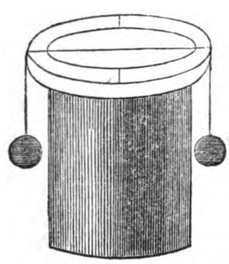
cards should afterwards be placed between leaves of clean paper, to preserve them from dust. Having obtained the fibres, they may be fixed as follows:—Take out the wire plate, remove the old wires, and clean the varnish from the engraved cuts. This must not be done by scraping with a knife, but by using spirits of wine and warm water, wiping the old varnish off, until the cuts appear quite clean. Now take a card, examine the fibre with a magnifying glass, and select a clean uniform piece.



NO. 4.

Take two little balls of wax, which attach to the extremities of the selected fibre, and cut off the remainder. If one of the balls of wax be held in the hand, and the other be allowed to hang freely, the fibre will become straight and untwined. Now take a camel hair brush, dipped in fair water, and rub the fibre gently, for the purpose of cleaning and damping it. Put the wire plate upon a little block of wood, *vide* fig. 6, and place the fibre upon it, taking care to examine with a magnifying glass that the wire falls into the proper cuts. The two little wax balls hanging on either side serve to stretch the thread, and keep it in its place. The cross wire is prepared in a precisely similar manner, and so on for as many fibres as may be required. Then, taking care that they are all truly adjusted in their respective lines or cuts, let a drop of varnish fall upon each cut, and put a tumbler over the apparatus, to cover it from dust. In twenty-four hours the varnish will have set, the ends of the fibres may then be cut off, and the wire plate carefully replaced in the telescope. This last is the most difficult part of the undertaking, because it requires very delicate handling to replace a wire plate without breaking a wire, the method of proceeding may however be acquired by patience and practice.

NO. 5.



NO. 6.

The best varnish to use is Copal varnish, but sealing wax dissolved in spirits of wine, friar's balsam, or laudanum, will answer. Fibres of raw silk are most suitable to small instruments, and this can easily be applied by following precisely the same rules. The previous damping of the fibres ensures their becoming tight and well stretched when dry. The best time to apply fresh wires is during the rains, when there is no dust flying about, and the atmosphere is rather damp. All the instruments should be carefully examined at that time, and those which require it should be fitted with fresh fibres. Freedom from dust also renders this season the most favorable for cleaning the axis and other working parts of the instrument which require to move glibly.

76. The next subject for consideration is the azimuthal microscopes, upon the proper adjustment of which the value of the azimuthal observation is greatly dependent.

77. When a circle is graduated into spaces of 5′ each, the rough reading of the degrees and fractional parts of the degree, as far as to the last 5′ space, is performed by an index having a single stroke upon a piece of metal, which is adjustable, and generally fixed on the clamp plate. The remaining minutes and seconds are shewn by the micrometer microscopes, of which there are generally three attached to the body of the instrument. These are firmly affixed to radiating arms, and are placed at equal intervals round the circle. The reading microscope, as now constructed by modern artists, is a species of compound microscope, consisting of three lenses, one of which is the object lens, and the other two form a positive eye-piece; the amplifying lens being omitted, as the field is not required to be extensive, and the measure is made near its centre. When firmly fixed and truly adjusted it is capable of sub-dividing the minute into seconds and fractions of a second with great accuracy and facility. It admits of the divisions on the limb being well illuminated, and does not injure them by friction, as happens with the vernier, over which it possesses all the advantages of convenience, combined with high magnifying power and micrometrical nicety of measurement. Its value, however, is entirely dependent on its being kept in proper working order and true adjustment, for which reason a knowledge of the principles of its construction is essential to an observer who aspires to great precision.

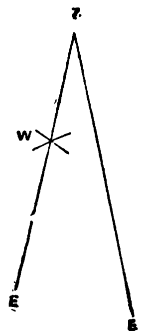
78. The arc usually measured by a microscope is a divided space of the value of 5′, to be sub-divided into 300″ by five revolutions of the micrometer screw, which therefore has its circular head sub-divided and numbered from zero by 10″, 20″, 30″, &c., up to 60″, every fifth stroke being longer than those of the other units, and terminating with a lozenge. Now, in order that the screw may be competent to measure a 5′ space without excess or defect, it is clear that the image of the latter must be magnified so as to occupy precisely the length of five revolutions of the screw. Whatever may be the focal length of the object lens, the image that it forms of an object (the divided limb of the circle for instance) will be more remote, or ascend higher into the body of the microscope the nearer it is brought to its solar focus, as measured from the limb. When it is placed exactly at solar focus, the rays passing through it become parallel, and the image will be formed at an infinite distance; but if the lens be placed beyond solar focus, so that an image is formed within the tube, then the two points where the divided limb and its image are situated are called conjugate foci, the latter of which recedes upward as the other approaches the object lens. If we call the distance of the limb from the lens f, and the distance of its image from the same lens F, the length of the image will exceed that of the object in the ratio of F to f,

or $\frac{F}{f}$ will represent the magnified state of the image. Hence it is manifest that the expression $\frac{F}{f}$ will have an increased value if we either augment F or diminish f. This is the fundamental principle to be attended to in adjusting the *runs*, viz.: If the object glass be protruded, it will approach nearer to the limb, whereby the run will be increased, and this will also be the case if the whole microscope be made to descend towards the limb.

79. Before describing the method of performing the adjustments, it may be necessary farther to premise that the microscope is in correct adjustment when the following conditions are strictly fulfilled, viz., when the image of the divided limb and the micrometer wires are so distinctly visible together that no parallax takes place by varying the position of the eye, in which state of good vision five revolutions of the screw must exactly measure one of the 5′ spaces of the limb, or ten revolutions measure two 5′ spaces.

80. To obtain distinct vision of the wires, place a slip of white paper on the limb, and illuminate it by means of the reflector, the wires will thus be projected on a blank field, in which state the eye is better able to judge of their distinctness. Now slide the eye-piece in its cell, until the wires appear sharply defined. This is a personal adjustment, and varies with the focal length of the observer's eye.

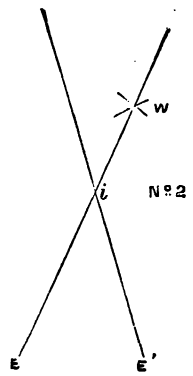
81. If, on removing the paper, the limb is not visible, the microscope must be moved bodily up or down,* till the limb is seen. This movement of the whole microscope is effected by releasing one and tightening the other nut, or collar-screw, which acts on the screw that is cut on the external cylinder of the microscope. One of these collar-screws is placed below, and the other above the plate, or collar, at the extremity of the arm upon which the microscope is supported, these collars being, in fact, the means of attaching it firmly to its support. Having rendered the lines on the limb visible simultaneously with the wires, bring one of the divisions to the centre of the field, and intersect it with the micrometer wire. Now move the eye to the left or right, and if the intersection remain steady, there is no parallax; but if the object appears to shift its



Nº 1

* It can easily be found out whether the microscope requires to be lowered or raised to produce distinct vision, because, if, on thrusting in the eye-piece, distinct vision is produced, there is far parallax, and the microscope must be lowered, and *vice versâ*. After this experiment the eye-piece must be carefully restored to the position adapted to distinct vision of the wires.

place, with respect to the wires, then notice in which direction it moves. If it follows the movement of the eye, there is distant parallax, which must be corrected by approximating the microscope to the limb. If, on the other hand, the object moves contrary to the eye, there is near parallax, which must be corrected by removing the microscope farther from the limb. This will be obvious from a consideration of the figures in the margin, of which No. 1 (page 26) illustrates the case of distant, and No. 2 of near parallax. For, let i be the image of a division, W the wires, E the position of the eye when both are seen together, and E' any other position of the eye; then it is clear that when the image is beyond the wires it will appear to move with E', but if the image be between the wires and the eye, it will move in a contrary direction to E'. This adjustment is performed by the same collar-screws described in the foregoing part of this paragraph, and is indeed merely the final perfecting of the adjustment for distinct vision of the limb.



82. The microscope having thus been freed from parallax, it will be necessary next to place a division of the limb 0, for example, exactly under the centre of the microscope, and then measure with the micrometer the number of revolutions and divisions between 359° 55′ and 0° 5′. If due care be taken to note the way in which the graduation on the micrometer head runs from 0 to 10″, 20″, 30″, it will be found that one cut or stroke on the limb will be arrived at by revolving the micrometer in the same direction with the numbering of the seconds on the head, while the other stroke will be reached by a motion of the micrometer contrary to the order of graduation. Subtract the reading of the latter from the reading of the former, and you will have the number of minutes and seconds measured by the micrometer. If this fall short of 10′ the magnifying power is too small, and the object glass must be protruded, in order to augment the run; but if the measured space exceed 10′, the object glass must be screwed in, to diminish the run. Now, in the former case, protruding the object glass will throw the image higher up the microscope, and produce near parallax, to correct which the whole microscope must again be bodily moved farther from the limb, and this again will diminish the run, so that the first correction should be a little over done to counteract the subsequent adjustment for parallax, and the whole process must be repeated two or three times, until the two conditions, with respect to run and freedom from parallax, are thoroughly satisfied.

83. It must now however be duly remembered that the divisions of the limb are themselves affected by error of graduation, to eliminate the effect of which it is desirable to take a mean of several 5′ spaces. Moreover, the plane of

the circle may not be exactly at right angles to the axis of rotation, in which case opposite parts of the circle will not be at equal distances from the object lens, except at one diameter of the circle, which will be a nodal line, and on all other diameters there will be a difference of run occasioned by the circle approximating nearer to, or receding farther from the microscope. It is also to be recollected that the excellence of the observations depends, not on a single microscope, but on the joint operation of three, so that although each may be near the truth, still the sum of their errors may be considerable; whereas, if the errors neutralised each other, the magnitude of the individual errors would not be of so much importance. To produce, therefore, the greatest degree of accuracy, the adjustment must be perfected with due regard to the foregoing considerations.

84. In general, with an instrument that has been previously in use*, the adjustments will be found already near the truth, and all that is required is occasionally an alteration of the run of a single microscope, so as to compensate the combined errors of the others. This principle has this advantage, that, inasmuch as the value of one micrometer has only the weight of one-third in producing the mean, so the correction to be applied to an individual microscope must amount to three times the sum of the errors of all the microscopes. For instance, if the mean value of three micrometers amounts to $10' + 2''$ for a $10'$ space, then $3 \times 2'' = 6''$ is the correction to be applied to a single microscope to reduce the sum of the whole to $10'$, and this correction is much easier applied than a smaller one to each.

85. The rule to be adopted is therefore as follows:—Set the instrument to 0° by the index, and a whole degree stroke will then fall exactly under the centre of each microscope. Now, as the micrometer in observing is limited to less than a $5'$ range on each side of zero, it is unnecessary to go beyond that extent on either side. It is also immaterial to read the whole degree stroke in the centre, because the sum of the $5'$ spaces will be given at once by the reading of the extremes; consequently, it will suffice to measure the $10'$ spaces under micrometers *A*, *B*, *C*, respectively, repeating each measure at least once, for the sake of precision, and to avoid mistakes. Now turn the instrument round to 180° in azimuth, and measure the $10'$ spaces as before. The mean value of each microscope will thus be obtained from the mean of $10'$ spaces at opposite diameters of the limb, and the mean value of the runs of all the microscopes will be got from six $10'$ spaces at every 60° of the limb apart, or from twelve independent $5'$ spaces, which should suffice. Now, if the mean error of the microscopes amount to more than $1''$ in $10'$, it will be advisable to correct that microscope which is most erroneous by a quantity equal to three times the combined error.† In per-

* Unless the microscopes have been dismantled, or taken off for the purpose of being cleaned, or of allowing the axis to be cleaned, or for any other similar reason.

† This correction, if the amount be only a few seconds, will not require the object

forming this adjustment the rules for avoiding parallax (as given in para. 81) ought to be strictly adhered to.

86. The following example will render this subject quite clear, and it is only necessary to add that the runs should be taken at every principal station, and duly recorded in the angle book, according to the form subjoined.

Example.

Observations to determine the runs of the Micrometers belonging to Harris' 15-inch Theodolite in use with the Karara Meridional Series, 19th April, 1845.

Reading of the Index.	Horizontal Microscopes. A. Lowest Reading.	A. Highest Reading.	A. Difference.	B. Lowest Reading.	B. Highest Reading.	B. Difference.	C. Lowest Reading.	C. Highest Reading.	C. Difference.	Remarks.
°	"	"	' "	"	"	' "	"	"	' "	The circle is divided into 5' spaces, and the runs were taken from 5' below to 5' above zero, the reading of the centre division being omitted as of no effect upon the means.
0	32·6	24·6	9 52·0	30·6	31·1	9 60·5	42·5	45·8	9 63·3	
	32·7	25·0	9 52·3	30·9	30·8	9 59·9	42·3	46·0	9 63·7	
180	30·2	27·9	9 57·7	34·0	36·1	9 62·1	25·2	22·5	9 57·3	
	30·0	28·2	9 58·2	34·3	35·9	9 61·6	25·4	22·2	9 56·8	
Mean of each,			9 55·05	..	..	9 61·03	..	..	9 60·28	
Mean of 3 Microscopes,			..	..	..	..	..	..	9 58·79	
Error on 10' run,			..	..	..	..	..	..	0 1·21	
Adjusted *A* Microscope to read about 3·6" more, after which the runs were as follows :—										
°	"	"	' "	"	"	' "	"	"	"	
0	20·5	16·4	9 55·9	40·5	40·9	9 60·4	52·4	55·9	9 63·5	
	20·6	16·3	9 55·7	40·8	40·9	9 60·1	52·5	55·8	9 63·3	
180	18·1	19·6	9 61·7	44·2	46·0	9 61·8	35·3	32·4	9 56·9	
	18·0	19·8	9 61·8	44·8	4.-1	9 61·7	35·3	32·3	9 57·0	
Mean of each,			9 58·73	..	..	9 61·00	..	..	9 60·18	
Mean of 3 Microscopes,			..	..	..	..	..	..	9 59·97	
Error on 10' run,			..	..	..	..	..	..	0·03	
Error on 5' run,			..	..	..	..	..	..	0·015	

glass to be disturbed, because it will generally suffice to raise or lower the whole microscope, until the required value of the runs is correctly obtained. The rationale of this proceeding is that the microscope is more sensitive, as regards the value of the run, than it is with respect to parallax, or distinct vision. On account of the spherical aberration produced by the lenses the exact position of the point of distinct vision is undefined to a small extent, and within this small limit the run may be sensibly varied without producing any appreciable parallax. This pecu-

87. The vertical microscopes are similarly adjusted, and their runs should be taken at every station, and duly inserted in the angle book according to the following form :—

Reading of the Index.	Vertical Microscopes. A. Lowest Reading.	A. Highest Reading.	A. Difference.	B. Lowest Reading.	B. Highest Reading.	B. Difference.	Remarks.
°	"	"	' "	"	"	' "	
F. R. 0	27·5	23·0	19 55·5	32·0	22·1	19 50·1	
	28·0	23·9	19 55·9	31·1	23·0	19 51·9	The vertical circle of this instrument is divided to every 10'.
F. L. 0	24·2	18·2	19 54·0	34·0	35·5	19 61·5	
	24·7	17·7	19 53·0	34·1	35·5	19 61·4	
Mean of each, ..	..		19 54·6	..	..	19 56·2	
Mean of 2 Microscopes,	..	..	..	..	..	19 55·4	
Adjusted *A* Microscope to read about 9" more, after which the runs were as follows :—							
°	"	"	' "	"	"	' "	
F. R. 0	15·2	19·7	19 64·5	20·2	11·7	19 51·5	
	15·6	19·8	19 64·2	19·8	11·5	19 51·7	
F. L. 0	12·8	15·6	19 62·8	21·0	22·3	19 61·3	
	12·5	15·1	19 62·6	21·4	22.5	19 61·1	
Mean of each, ..	..		19 63·5	..	..	19 56·4	
Mean of 2 Microscopes,	..	..	..	..	..	19 60·0	
Error on 20' run,	..	..	..	..	..	0·0	

88. The foregoing operations are very liable to disturb the lateral adjustment of the microscope, which is regulated by three screws, by means of which the microscope can be moved bodily a short space, so as to bring it over its proper

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liarity will perhaps be rendered clearer by applying numerical values to the expression $\frac{F}{f}$ which represents the magnified state of the image. Let $\frac{F}{f} = \frac{4}{1}$, and let the microscope be raised 0·01 of an inch, which will make $f = 1·01$. This will reduce the image in the focus in the proportion of 4 to 3·96, in which ratio the run will also be reduced, consequently as 4 to 300" so is 3·96 to 297·0". Therefore the descent or ascent of such a microscope will produce a variation of 3" for 0·01 of an inch perpendicular rise, while the latter quantity would hardly be perceptible in the shape of parallax.

division on the limb. After having, by means of these lateral screws, brought the microscope nearly over its proper division, the remaining minute portion of lateral adjustment can be readily effected by moving the comb, for which purpose there is a screw at the reverse end of the micrometer box. If the comb be moved or adjusted in the slightest degree you must be cautious to set zero of the micrometer head to correspond with zero of the comb. For this purpose take the milled nut of the micrometer head in one hand, and the graduated head in the other hand, then turn the latter round (taking care not to turn the micrometer screw also), and when zero on the head corresponds with the index on the micrometer, the adjustment is complete. The graduated head only retains its place by friction, therefore it is necessary to be careful in working the micrometer not to lay hold of the graduated head, but to use the milled nut which is given for that special purpose.

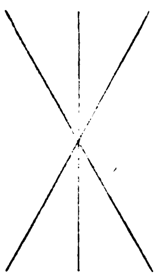
89. The microscopes should be fixed 120° apart, and should coincide with a whole degree stroke when the vernier is at 0°, but on account of the errors of graduation, it is advisable, in order that the minutes appertaining to microscope *A* may also answer for *B* and *C*, that the latter should be set to read 30″ or 40″ in excess, whereby the minute given by *A* will be common to *B* and *C*, provided care be taken to add 60″ to the readings of the latter when they fall into the next minute above. Suppose *B* and *C* are set to read 30″ and 40″ more than *A*, then if the reading of *A* is 0° 3′ 50″, that of *B* will be recorded as 80″, and that of *C* as 90″, which number of seconds are referable to 0° 3′. If this precaution be not taken the minutes, and sometimes the degrees, may require to be separately registered for each microscope, because, on account of errors of graduation, readings would frequently be obtained, as shewn in the following example, which would prove inconvenient for registry :—

A	*B*	*C*
0 0 10	359 59 50	359 59 55
62 11 5	10 45	10 55

90. The vertical microscopes are adjusted for parallax and run, in a manner precisely similar to the azimuthal microscopes, but, besides being set diametrically opposite to each other, they must also be freed from index error, that is to say, when the telescope is pointed to an object in the horizon the microscopes must be at zero, or if the telescope be pointed to an object of known altitude the microscopes must be set to give that altitude.

91. There remains yet to remark a few other points which should be attended to. For instance, it is clear that the micrometers should act in a tangential direction to the circle; therefore, if during the foregoing adjustments the direction of the microscope should by chance be changed, all that is necessary is to turn the microscope gently round in the collar until the micrometer is brought to act in the proper direction, when the collar-screws should be tightened, to

render its position permanent. If the wires form a cross, and the divisions on the limb are lines, then the tangential direction ought to be correctly attained, when the angles of the cross wires are truly bisected by the divisions, as shewn in the margin. Other instruments may be differently constructed, but the peculiarities of such cases will suggest the method of placing the micrometer in a tangential direction.



92. It is usual also at this time to take care that each microscope is set over the same part of the division lines, that is to say, the microscopes should be equidistant from the centre of the circle.

93. In measuring with a micrometer always use the right hand, or drawing motion of the screw, to finish an intersection, because, according to the principle of its construction, the micrometer screw acts in opposition to a spring at the reverse end of the box, the elasticity of which spring causes the shoulder of the micrometer head to press steadily on its proper bearing. When the micrometer head is released it would leave its bearing, and the screw would not be retracted, but for the action of the spring, upon the efficiency of which the uniformity of the motion of the micrometer during the releasing process entirely depends. It has however been found in practice that, owing to irregularities of friction, or other causes, the spring does not withdraw the screw uniformly and instantaneously. The head may be released by a small quantity before motion takes place. This is called lost motion, and occurs in all screws when this action is suddenly reversed. No reliance can therefore be placed on the action of a micrometer, unless the spring and screw are acting in opposition, whereby the bearing of the female screw is preserved, and uniformity and steadiness of action is maintained in immediate obedience to the rotation of the micrometer head.

94. It is not to be supposed, from what has just been remarked, that the micrometer is never to be turned to the left, that is to say, released; because it it is clear that, if constantly turned one way, the wire would soon arrive at the extremity of the field. The screwing up motion is indispensable only for the purpose of finishing an intersection. The rule is, to select that line or division which appears to fall nearest to zero in the micrometer, whereby the excursions of the wire are limited within a range of 5′. If the screw requires to be released, in order that the wire may reach the line which you intend to intersect, then all that is necessary is to overshoot that line, by carrying the wire about a minute beyond it, and then to reverse the motion. Complete the intersection by the screwing up rotation.

95. The graduation of an instrument consists either of lines or dots. The lines may be considered furrows, and the dots pits. It is obvious that if the light falls obliquely on the graduation, one side of the lines will be illumined,

and the other will cast a shadow. The apparent centre of the line, or dot, will, in this case, not coincide with the true centre, and if the graduation be coarse, the discrepancy produced may amount to two or three seconds. The obvious precaution to be taken is to throw light perpendicularly on the graduation, taking care that the field is equally illumined, and both sides of the division lines or dots are seen equally distinct, and sharply defined. The same care should be taken in illuminating the wires of the telescope, and as a general rule in observing angles. The same kind of illumination used in observing one station should also be used for the next ; that is to say that the two readings from which an angle is deduced should be taken under similar circumstances.

96. No good observations can be expected if currents of air are allowed to impinge on the instrument, or if it is subjected to the influence of solar rays, or any other causes calculated to disturb the equilibrium of the expansible materials of which an instrument is composed. Independent of this evil, it is also to be remarked that, if the wind is allowed to blow on the telescope, it frequently produces a lateral deflection to the amount of a few seconds. To protect the instrument from disturbing causes of this nature, it is sheltered by a tent, called an observatory tent, the observations being taken through small windows.

97. It is not however sufficient to be skilful and scrupulous in the manual and optical processes of observing; equal care and attention must be bestowed on the registry of the data in the angle books, otherwise the care and skill bestowed on the obervations will be entirely wasted.

98. An error in reading off a micrometer, or in writing down the reading, will produce just as great a discrepancy in the work as if the eye had made an error of the same extent in observing.

99. The observer should therefore give out the degrees and minutes in an audible voice, and the writer should repeat them while he is writing down. Unless this precaution be taken, errors may arise from peculiar pronunciation or defective hearing. Similarly, the readings of the micrometer heads should be given out distinctly to the nearest tenth of a second, and duly repeated by the writer. The figuring on the micrometer heads is seldom neatly executed, and a liability sometimes exists to mistake 30″ for 50″, and *vice versâ*, which if it occur in one microscope would affect an angle by nearly 7″, or the mean of the whole twenty-four angles by four-fifths of a second. Mistakes of this and similar kinds can only be avoided by strict attention. The writer should examine when the observations are repeated whether any reading is discrepant, and should give due notice to the observer of such an occurrence. The relative readings of the microscopes will generally serve to detect any error to the amount of a minute ; in the reading of *B* and *C*, for instance, if these be set to read always in excess of *A*, it will be necessary occasionally to add 60″ to their indications. The writer should always

take out the angles as soon as they are deduced, and insert them in a memorandum book, in a form which will enable the observer to see at a glance how the observations progress, and thus check any errors, either of commission or omission. Some observers take no heed of the results while they are engaged in observing, but make out the angles afterwards, a practice which cannot be too strongly deprecated.

100. The book in which observations are recorded is called an *angle book*, for a specimen of the mode of keeping which *vide* appended form. Two of these books are kept on each series, one of which is used in the observatory, and called the "Original Angle Book," the other is a transcript of the former, and is termed the "Duplicate Angle Book." Of these the former, when completed, will be transmitted by me to the India House, and the latter retained, as a record, in the Surveyor General's Office. The headings of the columns in the angle book are so explicit that no farther explanation seems necessary.

101. The angle book should on no account ever be suffered to fall in arrears. The original should be examined by two computers, and attested by their signatures, and the name of the observer should be recorded. The duplicate should be compared with the original by two persons, and attested by their signatures. It is a standing rule, in order to exclude errors, that all computations and comparisons should be performed independently by two persons, and attested by their signatures; and, unless such precautions have been observed, the results are considered untrustworthy as final work.

102. Observations should never be recorded on loose scraps of paper, or in pencil in rough books. The original record is always the most valuable document, and the labor of copying, besides being expensive, causes delay.

103. It is usual to add up all the angles observed at any one zero together, and divide them by the number, by which means you get as many sets of means as their are zeros, each mean being considered as an integral observation.

104. It is a fixed rule never to reject an observation, unless there be some obvious error in it. The circumstance of its differing, however widely, from the mean is not a sufficient cause for its expunction.

105. Having now remarked upon all the minutiæ to be attended to in the observatory*, it still remains to inculcate the necessity of the signals being care-

* It may here be remarked that in observing azimuthal angles the vertical clamp and tangent screw should never be used, as it is liable to deflect the telescope laterally. The telescope should be raised or lowered to the proper altitude of the object by the action of the hand only, the vertical clamp being left quite free. Similarly, in observing vertical angles it is unnecessary to clamp the azimuth circle; the vertical clamp and tangent screw should alone be employed.

fully adjusted to the station marks.* To insure this condition being fulfilled, the men who work the lamps and heliotropes should be trained to perform their parts with skill and fidelity.

106. Notwithstanding all the precautions which may be taken in making the observations, and recording them with fidelity and precision, it will be found that every mean angle is nevertheless burthened with small residual errors, which it is impossible entirely to eliminate, and which must therefore be dispersed in the subsequent office computations, agreeably to the rules which will hereafter be given. These errors arise partly from imperfections in the instrument, and in the observer, and some part of them may also be attributed to other circumstances beyond our control. For instance, the image of an observed object is subject to distortion, on account of the rays of light passing through a medium of very variable density and temperature.

106. There is no reason to conclude that the vertical refraction is all in the normal of either extremity of the trajectory; but, supposing this to be a rejectaneous consideration, still smoke and various vapours perpetually rising from the earth have probably not an uniform density at a given height above the surface, and the ray in its passage through these must unquestionably be liable to lateral as well as vertical refraction. We see, in fact, both by night and day that this cause is perpetually in operation; for the small disk of an Argand lamp, which is only twelve inches in diameter, and in a clear, settled atmosphere is reduced to a luminous point, swells out sometimes for several nights in succession into a broad ill-defined blaze, subtending occasionally two minutes of the horizon, and vibrating more like a sheet of fire than an object intended for accurate intersections, whilst the visible disk of the heliotrope, formed by limiting the rays to an aperture of two inches diameter, is even wilder and more straggling.

107. The only method of overcoming these sources of irregularity is to await a favorable state of the atmosphere, and be prepared to profit by it when it offers itself. Such occasions occur for day observation almost every sunny day for a shorter or longer period, between half-past three p.m. and sunset. The lamps are also frequently well adapted for observation from sunset to past midnight, especially in the early part of the season, but at other times they are steadiest from after midnight till about sunrise.

108. In hilly countries, when the stations are placed on lofty peaks, with deep intermediate valleys, objects are less disturbed by atmospheric causes than under other circumstances. Very good observations may then be obtained for an hour or two after sunrise; but, in general, however promising the apparent state of the atmosphere may be at sunrise it can rarely be depended on. It is at this period of the day that mirage is most conspicuous. Two or three heliotropes are

* A condition as indispensable to accuracy as the centering of the instrument itself. This subject will be adverted to under the head "Signal for Observation."

sometimes seen piled one above the other, or placed side by side, and then uniting together form columns or pillars of fire subtending several minutes, although produced by the rays of light passing through an aperture limited to two inches in diameter. This kind of lateral refraction is however too conspicuous to betray, but there is another insidious sort which ought carefully to be guarded against, because at the time it is in operation the visible disk of the lamp or heliotrope appears at first sight small, well-defined, and steady; if, however, it be carefully bisected on the wire, and then watched, it will be found after the lapse of a few minutes gradually to diverge to one side, and after a shorter or longer time will again move off slowly in another direction. This treacherous state of the atmosphere is very frequently in operation, and the only remedy is to watch carefully, and adjust the position of the telescope so that the object may appear to diverge equally on each side of the wire. With this precaution good observations can be made, but each intersection occupies several minutes of time, and the observations proceed very slowly.

109. The accordance, or otherwise, of angles taken at the same zero enables an observer to judge of the accuracy of his intersections, and the average errors of the triangles will show how far care and precaution have succeeded in eliminating the imperfections of the instrument, and other sources of error.

110. As perfect freedom from error is unattainable, success can only be judged from comparative approximation to the truth, and to enable an observer to form a fair estimate of his labors, compared with previous operations of approved merit, a table is subjoined shewing the errors of ten consecutive triangles, taken from the reports of the undermentioned Series.

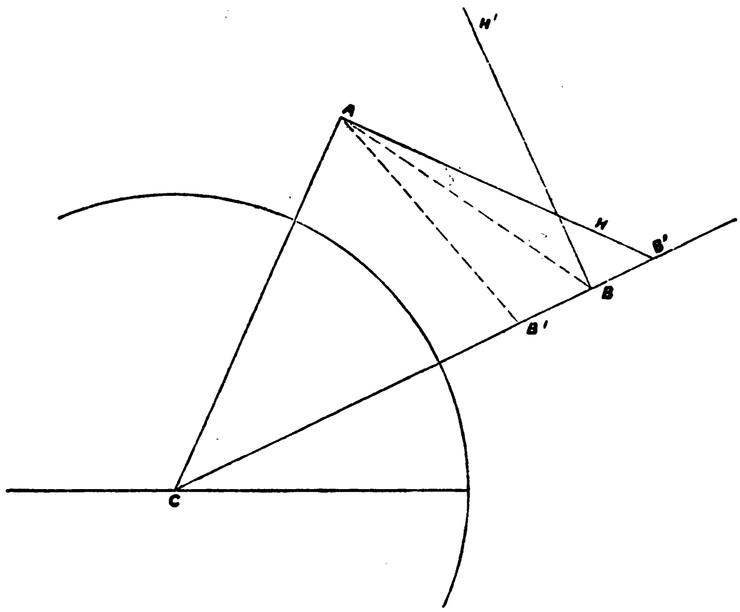
TABLE exhibiting the Errors of Ten Consecutive Triangles of the undermentioned Series.

No. of Triangle	Great Arc Series.	Amua Series.	Pilabit Terai Series.	North Long. Series.	Chendwar Series.	Bombay Long. Series.
	3-foot Theod. T. and S.	18-inch Troughton.	15-inch Cary.	18-inch Inst. by Col. Everest and Said Mohsin.	18-inch Troughton.	18-inch Dolland.
1	− 0·31	+ 0·71	− 0·79	+ 1·99	+ 1·52	− 1·04
2	− 2·75	+ 3·58	+ 2·07	− 0·45	+ ·84	− 0·39
3	+ 0·10	+ 0·07	− 2·97	− 1·12	− 2·36	+ 1·92
4	− 0·06	+ 0·23	+ 1·58	− 0·78	+ 1·64	+ 1·11
5	− 1·18	− 1·80	− 2·48	− 0·68	− 0·32	− 1·70
6	+ 2·72	− 1·21	− 3·92	− 0·39	+ 0·50	+ 0·11
7	+ 0·12	− 3·20	− 0·33	+ 4·41	+ 0·62	+ 0·24
8	+ 1·09	− 1·89	− 3·49	+ 0·02	+ 0·88	+ 0·34
9	− 3·32	+ 0·39	+ 2·19	− 0·88	− 0·36	− 0·03
10	− 0·05	− 0·42	− 1·61	+ 2·20	− 0·38	+ 2·98
	1·17	1·35	2·14	1·29	0·94	0·99

VERTICAL ANGLES.

111. The relative heights of the stations of a trigonometrical survey are deduced upon the following principles :—

112. Although the lines $B\ C$ and $A\ C$, which we will suppose to represent



the directions of plumb lines at B and A, do not really meet at C, the centre of the earth, (or even meet each other at all, unless under the same meridian or the same parallel of latitude,) yet the variation due to the compression of the earth may, in estimating the difference of height, be omitted as rejectaneous, and consequently $B\ A\ C$ will be a triangle, the angle at C being the angular distance between the two stations on the earth's surface. Now $A\ H\ B\ H'$ drawn perpendicular to the radii will represent the planes of the horizon, consequently though B is more elevated than A, still it may stand at an angle of depression. Let A and B be the angles of depression at A and B, then it is easy to see that

$$C + (\tfrac{1}{2}\,\pi - A) + (\tfrac{1}{2}\,\pi - B) = \pi,$$

or

$$A + B = C.$$

113. If one of these angles be an angle of elevation, the principle will remain the same ; for, by writing $-\ E$ instead of B, the formula is still $A - E = C$.

114. Suppose now that both A and B appear raised by refraction, and the

angles become $A - \delta A$, $B - \delta B$, or $E + \delta E$; then calling these new values A', B', or E', the equation will be transformed to

$$C - (A' + B') = \delta A + \delta B,$$
$$\text{or } C - (A' - E') = \delta A + \delta E.$$

115. It is usual to suppose an equality between δA and δB, and putting each equal to ρ, the mean terrestial refraction, we get

$$\rho = \tfrac{1}{2} \left\{ C - (A' + B') \right\}$$
$$\text{or} = \tfrac{1}{2} \left\{ C - (A' - E'.) \right\}$$

116. If an isosceles triangle be made by setting off $C\,B' = C\,A$, then $B\,B'$ will be the difference of height; also since $\angle\, B'\,A\,H = \tfrac{1}{2}\,C \therefore B'\,A\,B = \tfrac{1}{2}\,C - A = \tfrac{1}{2}\,(B - A) = \tfrac{1}{2}\,(B' - A')$ since $\delta A = \delta B$.

117. The angle $B\ A\ B'$ is called the subtended angle, because the difference of height is subtended thereby; call this angle S. Then if c be the length of the chord of the angle C, the difference of height will be deduced from the equation $\frac{c \times \text{sin. } S}{\text{cos. } B} = c \times \text{sin. } S \times \text{sec. } B = c \times \text{sin. } \tfrac{1}{2}\,(B' - A') \times$ sec. B, or $c \times \text{sin. } \tfrac{1}{2}\,(A + E) \times \text{sec. } B$. This equation is quite rigorous, and must be used when the depression is great, and consequently the difference of height considerable. For instance, when one station is on a lofty mountain, and the other in a low plain, the vertical angles may be very large, and the difference of height several thousand feet. But when A and B are small we may substitute unity for cos. B, and then the ordinary formula for small differences of altitude becomes $\delta h = c \times \text{sin. } S$.

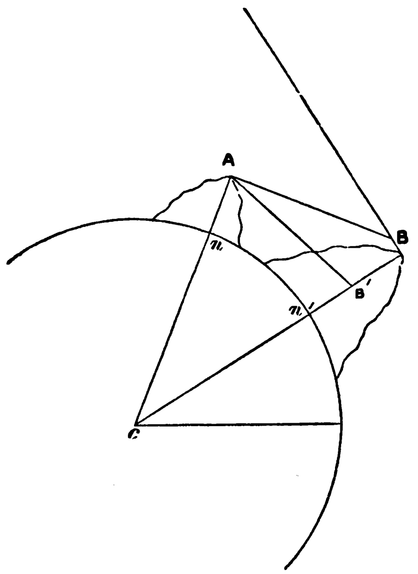
118. Several other considerations must now be made. The angles at both A and B are observed with an instrument which is elevated by a quantity a above the point whose height is sought. The corresponding angular value, therefore, or $\frac{a}{c \text{ sin. } 1''}$, is a correction to be subtracted from all angles of depression, and added to all angles of elevation.

119. It sometimes happens that the vertical angles are taken to the top of an insufficient platform, which after being visited is raised; let the absolute height thus added be p, then $\frac{p}{c \times \text{sin. } 1''}$ is a correction which is to be subtracted from all angles of depression, and added to all angles of elevation observed prior to the augmentation, in which case the new platform is treated as part of the mountain.

120. Daylight angles are often taken to the top of the flag-staff, or to an

object raised above the station mark, in which case $\frac{f}{c \sin. 1''}$ is to be applied positively to all depressions, and negatively to all elevations, f being the height of the object or staff.

121. Lastly, c to be accurate should be corrected for the height of the given station (A) above the sea, because c is the geodetic distance, $n\,n'$ given in the triangle sheets, whereas the true distance $A\,B'$ is the element required in these computations. Therefore let h be the height of the given station (A) above the sea level, and r' the radius of curvature;



then $r' : r' + h :: c : c + \delta c$, or

$r' : h :: c : \delta c = \frac{c\,h}{r'}$, in which c may, as before, be taken for the measured chord, or measured arc, as given in the triangle sheets.

122. But it is the logarithm of $c + \delta c$, and not the natural number δc, that is required in the computations, and it will be more elegant, as well as expeditious, to deduce the correction to the log. of the geodetic distance. This correction is always additive, and may be found from the following formula :—

$$\text{Ar. Co. log. } r' + \text{log. } M + \text{log. } h,$$

the natural number answering to which, carried to 7 places, is the correction required for log. c. The term M of the foregoing formula is the modulus of the decimal system of logarithms.

123. The problem of determining the relative heights of two stations A and B rests upon the foregoing mathematical principles, and the practical part of the operation obviously consists in carefully measuring with the instrument at each station the altitude or depression of the others.

124. It will be recollected that in art. 115 it was assumed for granted that the refraction at both stations is the same; but, in order that this equality may become a probable occurrence, the back or reciprocal vertical angles ought, rigorously speaking, to be taken at one and the same instant.* But as that

* This was done on the Great Indian Arc, in crossing the plains of Hindostan, because the vertical angles were there subject to the influence of extraordinary refraction and mirage, on account of all the rays grazing along the surface of the ground.

cannot always be managed, the next best method is to be careful to observe them about the same hour from noon, for then the errors caused by terrestial refraction are more likely to compensate each other. The best period of the day for observing verticals is the time of minimum refraction, which usually occurs about 2-45 to 3-45 in the afternoon. Long experience has proved that reciprocal vertical observations taken to heliotropes at the period of the day above indicated give the most accordant results, and may therefore be confidently relied on. If the stations are not visible at the time of minimum refraction, the precaution should be taken of observing them simultaneously, by means of two instruments and two observers.

125. Four sets of observations, two with the face each way, are sufficient at each station, and every occasion should be used when the rays cross each other and form separate series to get as many points of verification as offer themselves.

126. Vertical observations taken with the face each way, that is to say, with the face of the vertical circle in one instance to the left hand, and in the next to the right, are called collimated observations, because the mean of the two faces gives a vertical angle free from error of collimation, which is not the case with a single face.

127. The method of observing is as follows :—The instrument should be duly levelled and adjusted previous to the time of minimum refraction; and as soon as objects are steady, or nearly so,* direct the telescope to one of the principal stations, and having clamped the vertical circle, bisect the object with the horizontal wire, and note the hour and the minute at which the observation was made. Now read off the microscopes of the vertical circle, according as the vertical angle may be an elevation or a depression, and let the readings be duly recorded in the angle book; release the clamp; turn over the telescope 180° in altitude, and the instrument round 180° in azimuth; bisect the same station again, and record the readings as before, together with the hour and minute of time at which the bisection was made. The mean of the two observations will give one collimated vertical angle. Repeat the last observation as before, and then reverse the instrument, and take another observation, which will complete the second collimated vertical angle. If these two be accordant they will suffice, otherwise a third, and even a fourth, ought to be taken. Now direct the instrument to another station, and take the vertical angles in the same manner, and in that way proceed until all the surrounding stations have been observed; but if

* Heliotropes are seldom perfectly steady till 4 or 4½ p.m., but at that period they are beginning to rise at the rate of a second per minute. Although they may appear a little agitated at 3 p.m., still the refraction is then least, and the observations give the best results. In fact, any error arising from the agitated aspect of the heliotrope will be much less injurious th n refraction, and indeed may be counteracted in a great measure by multiplying the observations, because the agitation of the disk does not produce a constant error in one direction, as happens in the case of uncertain refraction.

the objects begin to rise, which will be ascertained from the depressions diminishing, or altitudes increasing, it will be necessary to desist for that day, and reject any observation which may have been made after the time of minimum.

128. When you arrive at a station which has been observed from one in rear, the back or reciprocal observations must be taken within a minute or two of the corresponding times. If any atmospheric change should appear to take place intermediately to the observations of the reciprocal vertical angle, the equality between the refractions, which the nature of the problem demands, is likely to be destroyed, and therefore too long a space of time ought not, if possible, to be allowed to elapse before the reciprocal observations are made.

129. The example given in the appendix shews the method of recording the observations in the angle books, with reference to which it will be seen that on both faces they are registered as depressions or elevations. It must however be borne in mind that if the instrument gives altitudes on one face, it will give zenith distances on the other, and the graduation on the micrometer head will, in this case, be according to the order of the figuring; consequently, the system of reading altitudes on one face, and zenith distances on the other, is, perhaps, the safest course to pursue. In the example, however, altitudes were read on both faces, by taking the seconds from the micrometer head in an inverse order, thus 50″ was read 10″, 40″ was read 20″, and so on, which gives altitudes instead of zenith distances. This was the method followed on the Great Arc, and several other Series, and with care and experience it may be practised with facility, and without liability to mistakes. With regard to depressions it may also be remarked that the face of the instrument which gives altitudes according to the direct order of graduation, will give nadir distances in the same direct order for all objects below the horizon; and on the opposite face, which gave zenith distances according to direct order of graduation, depressions may be read off in direct order. Instead of nadir distances, however, the depression may be read off at once by supposing the divisions on the micrometer head reversed, viz.: 50″ = 10″, 40″ = 20″, &c., as in the case of altitudes.

130. Those instruments which give altitudes and nadir distances in direct order on one face, and zenith distances and depressions on the other, are numbered in four quadrants from 0° to 90°, following each other round the circle; that is to say, 0°, 10°, 20°, &c., to 90°, 10°, 20°, &c., to 90 , 10°, 20°, &c., to 90°, 10°, 20 , &c., to 90°; the last 90 being the zero of the first quadrant. Other instruments however are divided from 0 at the eye and object end to 90° both ways. These figures, as far as the circle is concerned, give altitudes and depressions on both faces; but the ordering of the graduation on the micrometers is only suitable for one face, and must be taken in reverse order on the opposite face.

131. On account of the variety of graduation which obtains in different instruments, no fixed rule can be laid down, and the peculiarities of each must

therefore be studied carefully by the observer. Those in which the vertical circle is less than fifteen inches have microscopes that read to 2″ only, one revolution of the screw being equal to two minutes on the limb. These require even greater care and attention in reading off than others.

132. The following rules obtain in all instruments :—For altitudes the micrometer should be moved to the division apparently above the zero of the micrometer at the eye end, and to the division below zero of the microscope at the object end. It is, of course, exactly *vice versâ* with respect to depressions ; that is to say, the divisions which appear below zero at the eye end, and above zero at the object end, should be chosen. Now, if on revolving the micrometer head, so that the wires shall proceed from zero in the microscope to the divisions above indicated, if it be remarked that the graduation on the micrometer head follows in the natural order of numbering, then the reading must be taken direct, otherwise it must be made reverse. After studying each instrument, a rule can generally be framed like the following, which applies to Troughton and Simms' great theodolite, and I believe also to the 18-inch instrument by the same makers, viz.,—

F. L. { Elevations read direct.
Depressions read reverse.

F. R. { Elevations read reverse.
Depressions read direct.

A rule such as this having once been carefully deduced for the particular instrument will prevent subsequent mistakes.

133. The same precautions which are necessary in adjusting and reading the azimuthal microscopes are also indispensable in using vertical microscopes. They should be freed from parallax, and adjusted for runs ; they should also act in the direction of a tangent to a circle, and all measurements should be made with that motion of the screw which draws against the spring ; moreover, they ought to be set truly horizontal, with respect to the axis of azimuth motion and the division of the vertical circle, as they will then be free from index error, or nearly so.

134. Although the mean of two observations with the face each way gives a result independent of collimation or index error, still, when the latter is large, much inconvenience is felt. All that is necessary to free the instrument from this error, or rather to render that error very small, is to take the altitude or depression of any fixed terrestial object on both faces, and deduce the mean, which will be the true vertical angle. Then, while the telescope remains firmly clamped, with the object truly bisected on its horizontal wire, move the microscopes by their appropriate screws, till they give the reading indicated by the vertical angle deduced from both faces. There are three screws which act through the collar at the extremities of the radiating arms, which screws, when the collar-screws are a little relaxed, enable the microscopes to be moved through a small space laterally ; therefore, having set the graduated head to the reading required, move the whole

microscope by means of the screws above mentioned, until the cross wires are over the proper division on the limb; then tighten the collar-screws, in order that the adjustments may remain permanent. The lateral screws, however, will not enable you to set the microscope much nearer than a minute, and the remaining portion of the adjustment must be effected by altering the graduated head of the micrometer, and then setting the comb to correspond therewith, as described in art. 88, when describing the adjustments of the azimuthal microscopes.

135. In arts. 118 and 120 it has been remarked that corrections must be applied for the height of the instrument, and also for the height of the observed objects. The former should be carefully measured by the observer, and the latter by the men in charge of the heliotropes and lamps, so that on arrival at each station the heights of the objects may be duly measured, and noted in the form subjoined.

Eye Stations.			Object Stations.			
	Height of			Height of		
Name.	Marks above Ground.	Instruments.	Name.	Heliotrope.	Lamp.	By whom Measured.
	Feet.	Inches.		Inches.	Inches.	
Imlia,	0·0	69·75	Asrofpur.	26·	..	Ramdeen.
	8		Thana.	30·	..	Jeri.
	24		Ragaopur.	82	..	Bikari.
			Samnadeo.	30	..	Manbod.

136. Hitherto we have only considered the case of principal stations at which the back, or reciprocal, vertical angle can readily be observed; but it is clear that if the amount of refraction were known, the relative heights could be deduced from verticals taken at one extremity of a ray, whereby the height of intersected secondary points could be ascertained.

137. It is usual to estimate the terrestrial refraction in terms of the contained arc, that is to say, $\rho = n\,C$, whence $n = \frac{\rho}{C}$, and by the supposition of an equality between the refractions at the extremity of a ray we obtain the value of n from all the observations at every principal station by the formula in art. 115.

138. The mean of all the values of n at any station may therefore be adopted as a mean ratio of refraction to contained arc at that locality, and this mean refraction may without risk be applied to correct any single vertical angle taken to any secondary point; consequently, if D be the observed depression of that secondary point, and C the contained arc due to its distance, then $D + n\,C = \Delta$ will be the true depression cleared of refraction. Let the unob-

served reciprocal vertical be D' or $-E'$, its corresponding value in vacuo being Δ, then as $C = \Delta + \Delta'$, therefore $\Delta' = C - \Delta$, and the subtended angle $S = \frac{1}{2}(D - D') = \Delta - \frac{1}{2}C = D + nC - \frac{1}{2}C$, or for an elevation $S = E - nC + \frac{1}{2}C$.

139. Upon this principle may be deduced the relative height of a secondary point from a vertical angle observed at one station only, without any corresponding back observation; but, in order that the factor n may be used with confidence, the vertical should be taken about the same time from noon as the other verticals from which the value of n is derived. Some observers adopt a mean refraction deduced from the whole series of triangulation, or they employ the factor deduced from other geodetical operations of known celebrity, such as the English and French trigonometrical survey operations; but the factor obtained from observations made on the spot appears preferable, as better adapted to existing circumstances of locality and atmosphere. It may be premised, *à priori*, that refractions observed in the climate of France or England are not likely to assimilate with others made in the torrid zone, nor is the refraction at a station little elevated above the sea likely to be the same as the refraction on a lofty mountain, where the barometric pressure and temperature are so much smaller.

140. Every consideration therefore seems to combine in favor of the adoption of a local value of the factor of refraction.

141. If a secondary station be observed from several principal stations situated at very various distances, the refraction can be inferred by another process, viz.: by assuming a variety of different values for n, and finally adopting that value which furnishes the most accordant results. This method was adopted by Mr. H. Colebrooke, in determining the height of the mountain Dhawalagiri, vulg. Dhologir, situated to the north of Nepal. The following extract from his paper is taken from Vol. 12 of the "Asiatic Researches":—

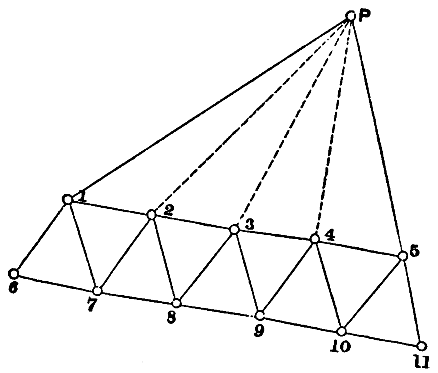
Station.	Distance in Miles.	Intercepted Arc in Degrees.	Altitude by Observation.	Height, allowing for Refraction.						
				$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{14}$
		° ′ ″	° ″							
A	89·35	1 17 51	2 48	24875	26663	27110	27476	27558	27626	27855
C	102·85	1 29 36·6	2 19	24348	26716	27308	27792	27900	27991	28294
D	136·35	1 58 18	1 22	21338	25494	26554	27384	27573	27773	28286
			Mean,	23520	26091	26784	27551	27677	27797	28145
		Extreme Difference,		3537	1222	774	408	342	365	439

"It is apparent from inspection, that the observations at the stations *A* and "*D* agree best; and if that computation be nearest the truth, wherein the ex-

"treme differences are least, the conclusion will be, that the height is about "27,550 feet; such being the elevation deduced from the mean of observations "calculated according to middle refraction."

142. The lofty snowy peaks situated north of Nepal are the most stupendous pinnacles of the globe. Their heights and relative situations are therefore most interesting desiderata in general geography, and should form prominent objects in the geodetical operations now in progress in Bengal.

143. The positions of the snowy peaks can be laid down with the greatest chance of success from the stations of the North Longitudinal connecting Series, which, as it approaches nearest to them, and follows a parallel direction, will afford the longest bases for determining their distances. Their heights may also be deduced with the greatest accuracy from the vertical angles taken at the shortest distance from them; but as the refraction adopted in reducing the observation will form an important element in the computation, it is desirable that vertical angles should be taken at a great variety of distances, which will afford a valuable check on the results. For instance, if *P* be a snowy peak, and 1, 2, 3, 4, 5, &c., be the stations of the North Longitudinal Series, then the position of *P* will best be determined by the triangle 1, 5, *P*; but the azimuths at 2, 3 and 4 will also be valuable, as establishing the identity of the point observed from 1 and 5.* The vertical angles at 2, 3, 4 will be equally valuable with those at 1 and 5, while those at 16, 15, 14, 13, &c., will serve the purpose of verifying the factor of refraction.



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* In observing unmarked points there is always a risk of mistake, because the appearance of a peak varies so much in different aspects, and the only way of efficiently checking the identity of the point is to take frequent observations at short distances apart.